

Ideas in Mathematics, Fall 2023, Weekly worksheet 4

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1. Prove that any positive integer greater than or equal to 9 can be written as a sum of the form $a + b$ where a is a multiple of 5 and b is even.

2. Suppose a has a remainder of 8 when divided by 12 and b has a remainder of 7 when divided by 12. What will be the remainder of ab when it is divided by 12? Explain why this is true

(you don't need to give a formal proof, but give an explanation based on the ideas from class).

3. Suppose a has a remainder of 1 when divided by 73. What will be the remainder of a^{27} when divided by 73?

4. Find the last digit of the following numbers (in their standard base 10 decimal expansions). Give some explanation for your answers.

(a) 231^{297}

(b) 99^{100}

(c) 77^{99}