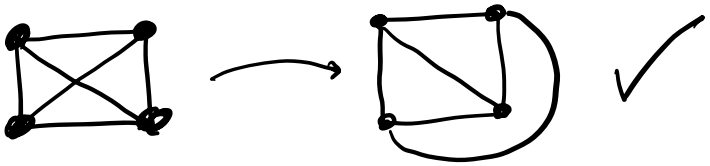
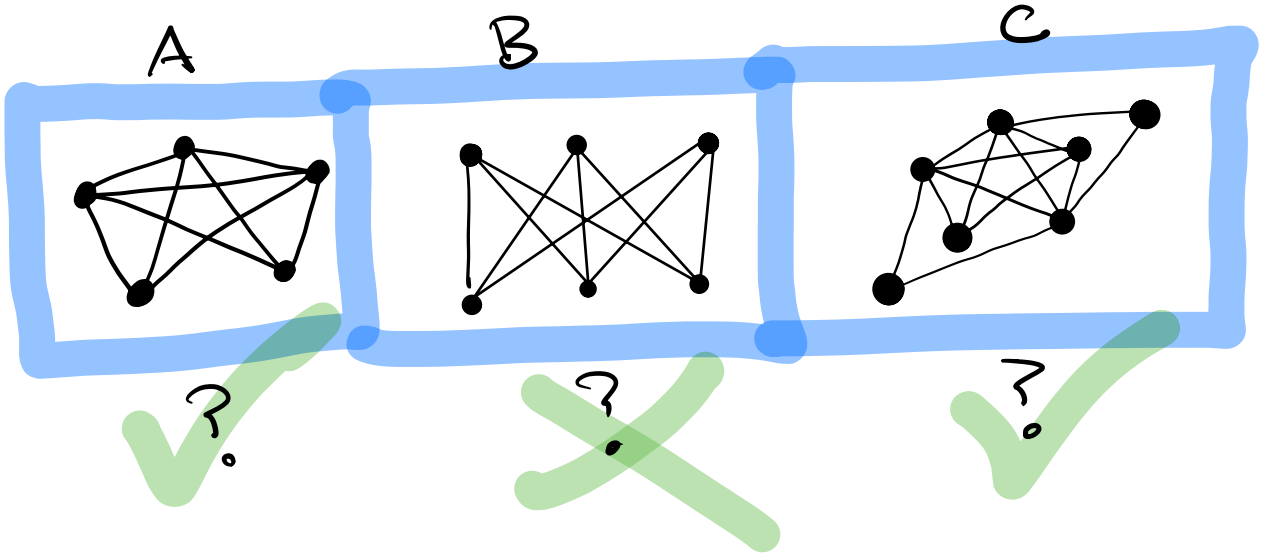
Which of these can be redrawn so the same dots are connected but with no lines crossing?

example  ✓



Graph theory

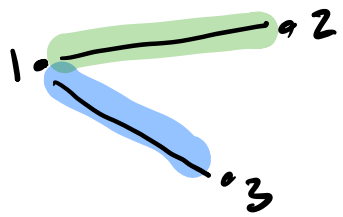
What is graph?

Formal definition: A graph is a set V whose elements we call "vertices" and a set E whose elements are subsets of V with 2 elements each which we call "edges"

Ex: $V = \{1, 2, 3\}$ $E = \{\{1,2\}, \{1,3\}\}$

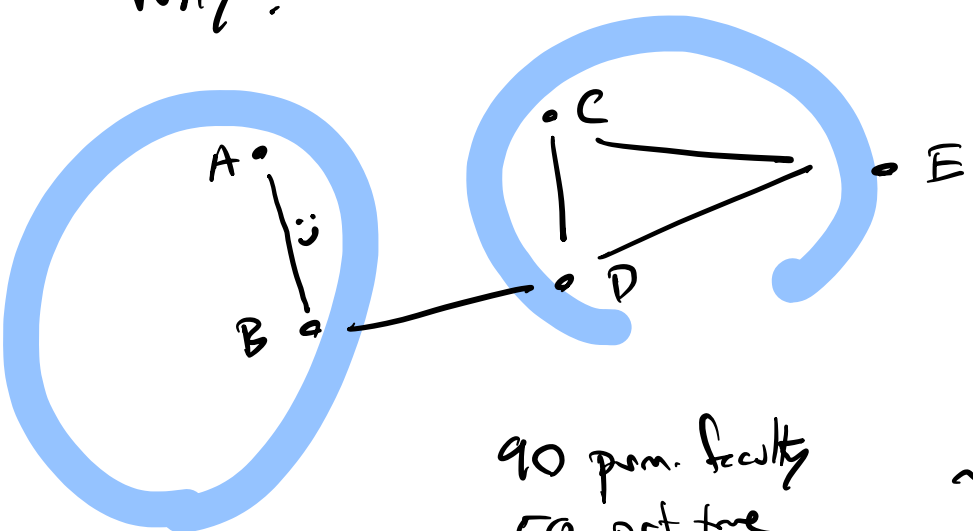
The natural numbs is a set $\mathbb{N}$ w/ an elemt "0"
and a function $S: \mathbb{N} \to \mathbb{N}$ such that...

→ Pictures



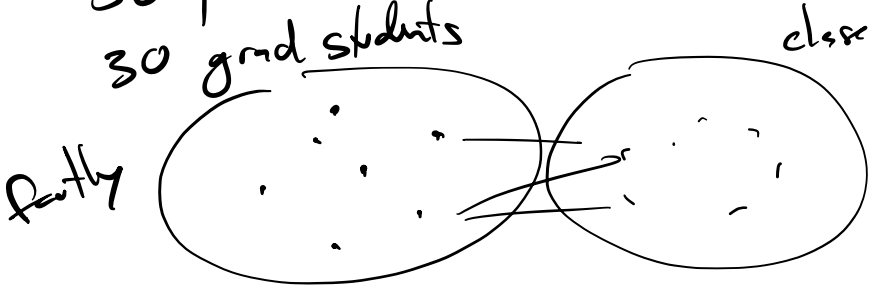
$= G$

What ✓
Why? → networks (social / streets correctly restructures)

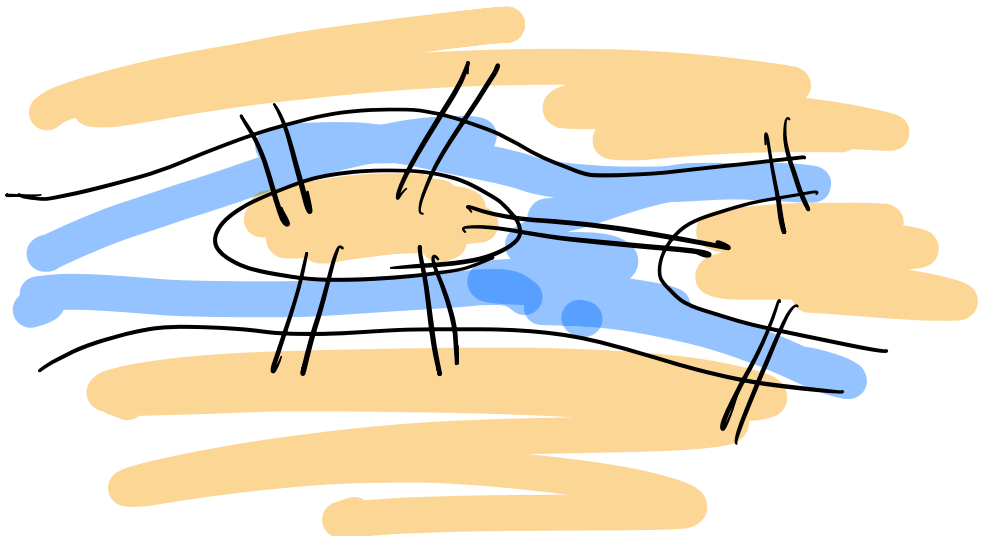


90 prim. faculty
50 part time
30 grad students

~400 classes.



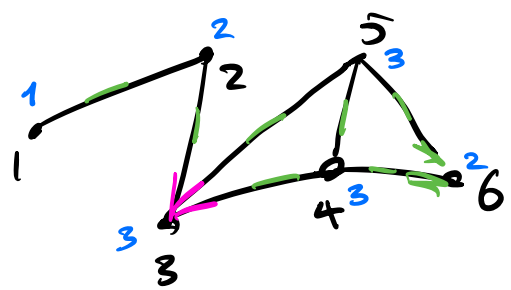
- circuit design
- data structures



Fundamental results of graph theory.

If G is a graph with edges E & vertices V

Definition the degree of a vertex is the # of edges incident to it.



$\deg(\text{vertex } 3) = 3$

$\deg(\text{vertex } 6) = 2$

Degree formula the sum of the degrees = twice the edges

14

2 × 7

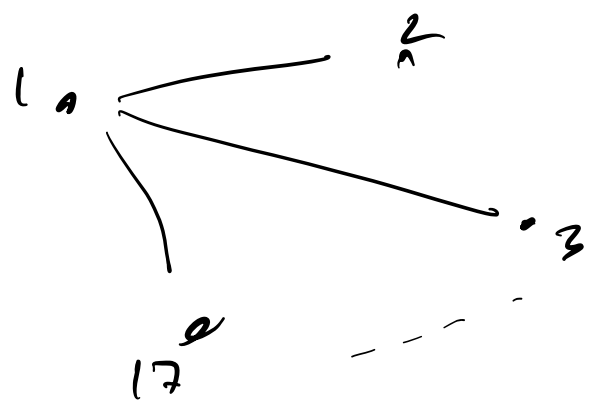
Explanation:

each edge contributes 2 to the total degree count

✓

Suppose 17 people.

is it possible for each to shake hands w/ exactly 3 people?



$\sum \deg(v) = 51$

$= 2 \times$ #edges

↑ handshakes.

☹

25.5

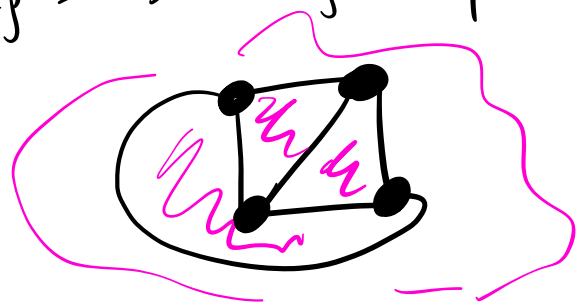
X

Definition A graph is called "planar" if it can be drawn without edges crossing.

ex 

Fundamental observation:

drawings of planar graphs break up the plane into regions



$f = 4$
$e = 6$
$v = 4$

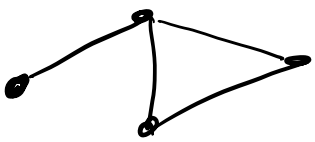
Euler:

observed: if we have a planar graph w/

$\#V = v$ $\#E = e$ $\#\text{regions} = f$

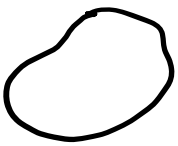
then $f - e + v = 2$

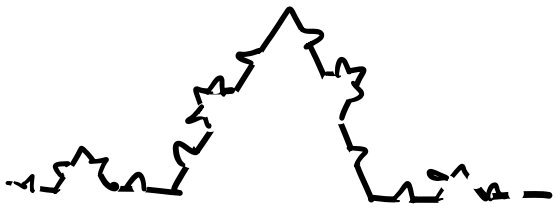
$4 - 6 + 4 = 2$

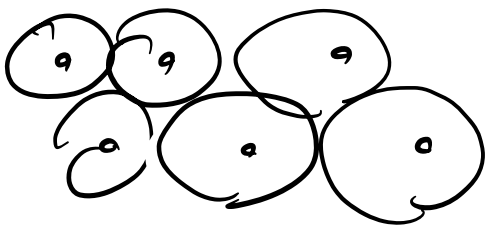


Segway: "take nothing for granted"

"Jordan curve theorem" "a simple closed curve"
has an interior & exterior region







Application