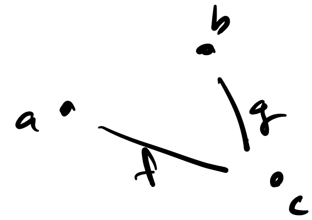
Banff Candy Store "Mega Sour"

Last time: Started graph theory



Recall:

A graph G consists of

- A set of "vertices" $V = V(G)$ $\quad V = \{a, b, c\}$
- A set of "edges" $E = E(G)$ $\quad E = \{\{a,c\}, \{b,c\}\}$ (f, g)

  represented by sets consisting of 2 vertices.

If $v \in e$ we say v is incident to e

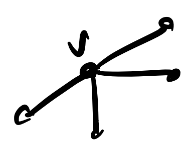
If $v, w \in e$ we say v & w are adjacent.

$\#A = |A|$ number of elements in set "cardinality"

Basic Notions

Def the degree of a vertex v is defined as

$$\deg(v) = \#\{e \in E(G) \mid e \text{ is incident to } v\}$$

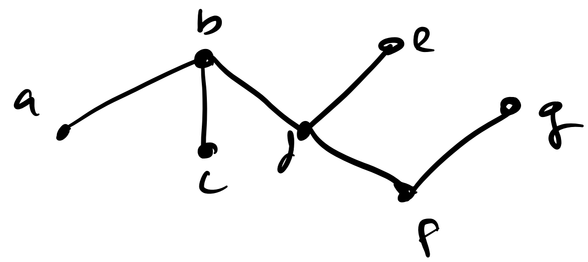


$\deg(v) = 4$

Recall: The degree formula

$$\sum_{v \in V(G)} \deg(v) = 2\#E(G)$$

Def if x & y are vertices, a walk W from x to y is a sequence of vertices $W = (u_1, u_2, \ldots, u_n)$ $u_i \in V$ such that $u_1 = x$, $u_n = y$ & u_i is adjacent to u_{i+1}



$W = (a, b, d, f, g)$ is a walk from a to g

$= (a, b, c, b, d, e, d, f, g)$

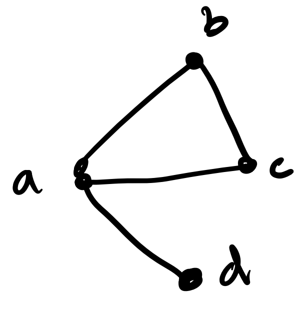
if $x = y$ (begin = end) we say W is <u>closed</u>

if $x \neq y$ we say W is <u>open</u>

If $W = (u_1, \ldots, u_n)$ is a walk, the edges of the walk are $(\{u_1, u_2\}, \{u_2, u_3\}, \ldots)$

$\{u_1,u_2\} = e_1$, $\{u_2,u_3\} = e_2$, ...

If edges are all distinct, we say the walk W is a <u>trail.</u>

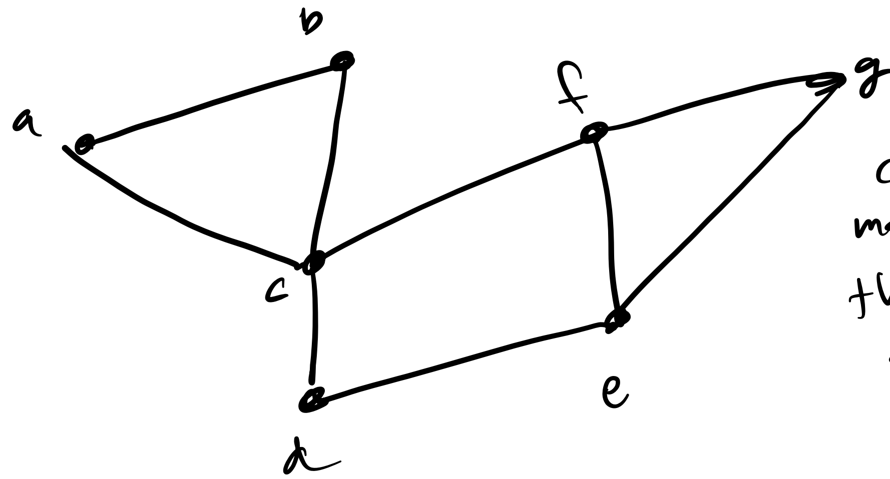


$W = (a, b, c, a, d)$ is a trail from a to d

$W = (a, b, a, d)$ is not a trail

$\{a,b\}, \{b,a\}, \{a,d\}$

a closed trail is called a <u>circuit</u>.

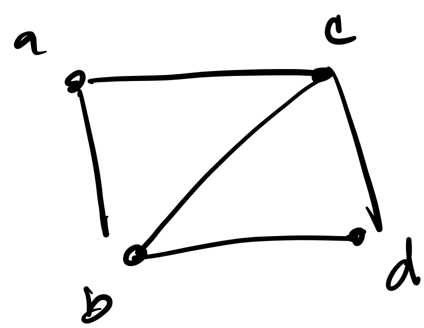


can we make a <u>trail</u> that passes through every edge?

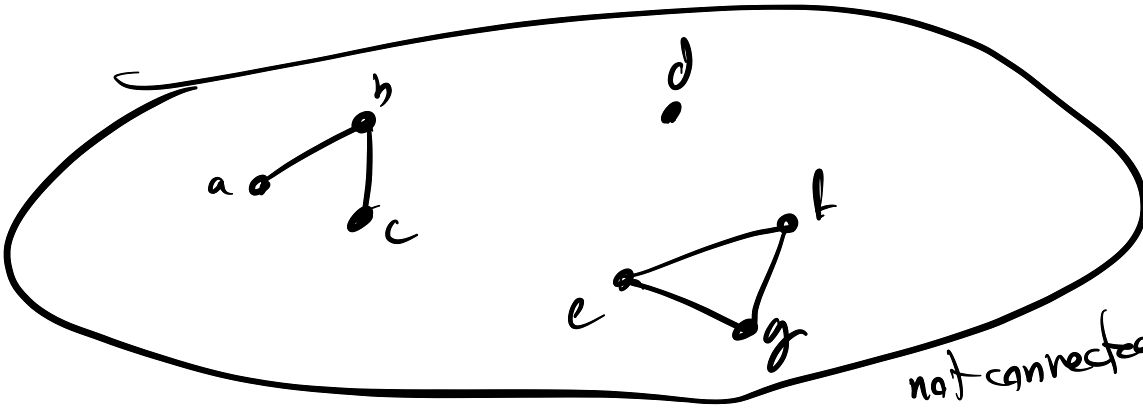
If a walk has no repeated vertices, it is a path

a "closed path" is called a cycle

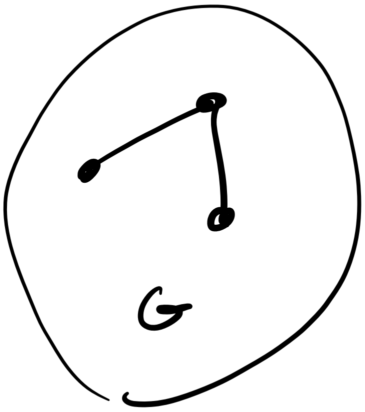
(i.e. only repeat = begin & end)



<u>Def</u> A graph is <u>connected</u> if there is a walk between every pair of vertices.



not connected.



$V = \{a, b\}$
$E = \emptyset$

Lemma If G is connected then there is a path between any two distinct vertices.

In other words: if there is a walk between any two vertices, there is also a path.

Proof: Let $x, y \in V$. Want to show there's a path from x to y.

Consider all possible lengths of walks from x to y. This is a set of natural #s.

Therefore there is a shortest (smallest length) of walk.

Let $W = (u_1, \ldots, u_n)$ be a walk from x to y with shortest possible length.

<u>Claim:</u> W is a path

why? if it wasn't, we'd have

$$u_1, u_2, \ldots, u_i, u_{i+1}, \ldots, u_{i+j}, u_{i+j+1}, \ldots, u_n$$

(u_i and u_{i+j} joined: $=$)

$u_i = u_{i+j}$ some i, j.

but then $u_1, u_2, \ldots, u_i, u_{i+j+1}, u_{i+j+2}, \ldots, u_n$ would be a shorter walk (length $= n - j$)

but W was shortest possible length, so impossible!

so W is a path. ∎

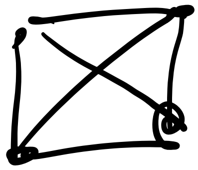
remove

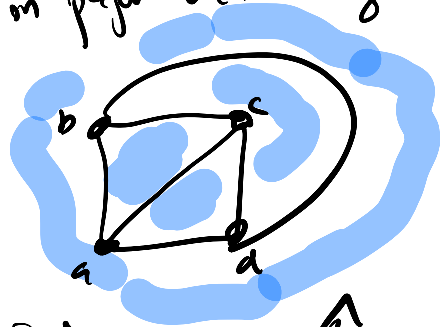
$u_1, u_2, u_3,$ ~~u_4, u_5, u_6~~$, u_7$

$i = 3 \quad i + j = 6 \quad j = 3$

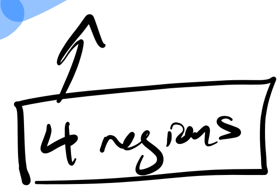
Planar graphs

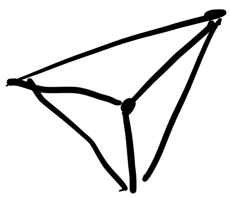
a graph you can draw on paper w/out edges crossing.

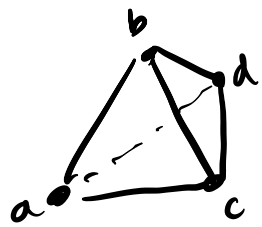


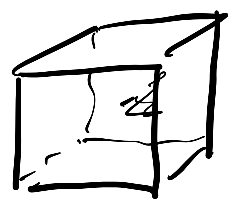


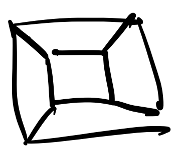
divides the plane into regions

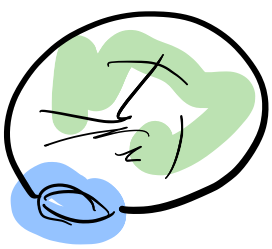


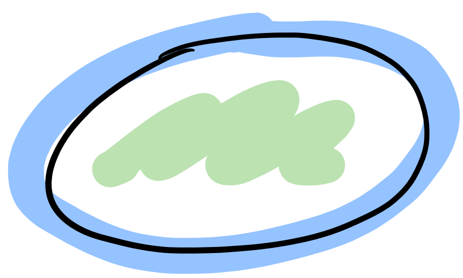


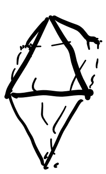


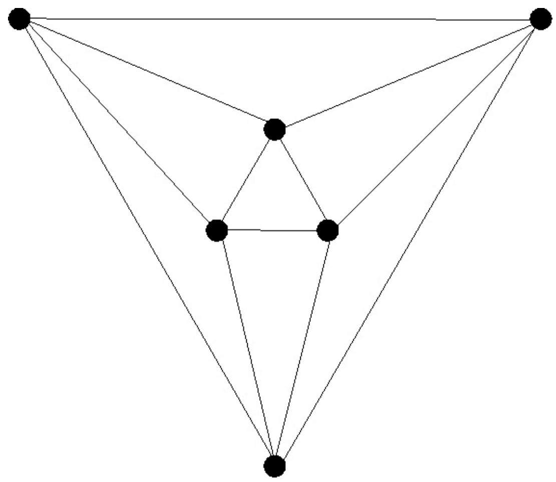


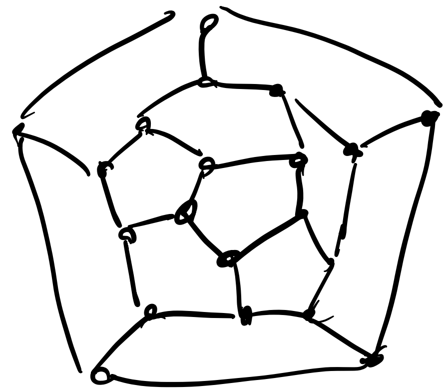






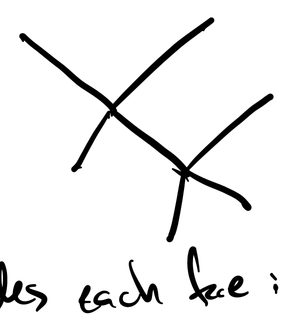


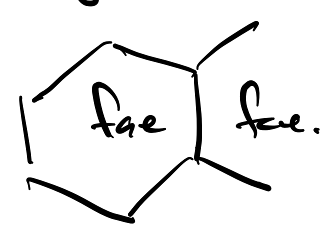




to classify the platonic solids,
we need to describe their possible graphs.

these have some # of faces f all same # of sides k
and each vertex should have same degree m





f faces, k sides each face: graph has

each edge $\longleftrightarrow$ 2 faces

$\frac{f \cdot k}{2} = e$

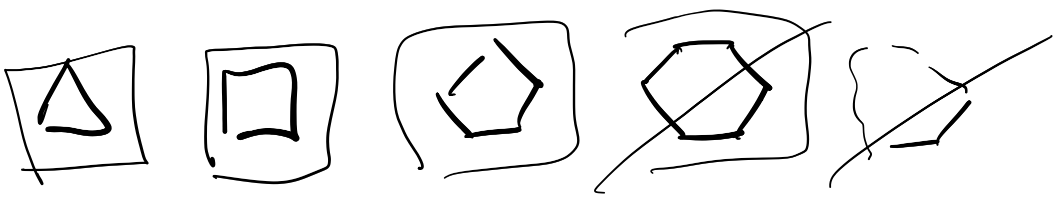
each face contributes to k edge sides

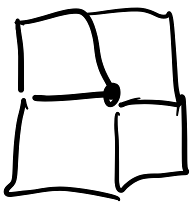
total contribution of all faces $f \cdot k = 2e$

Euler's formula

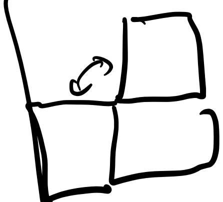
#faces − #edges + #vertices = 2

how many polygons around a pt.

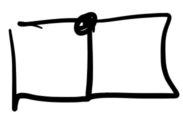




4X



3 □'s.



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