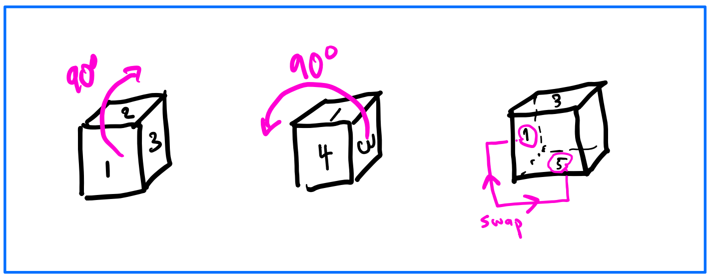
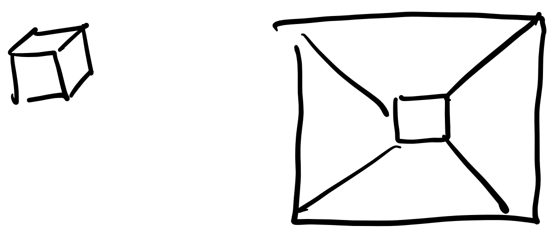
Puzzler

take a cube with sides labelled, turn it once over away from you, then once counterclockwise. then switch the labels on the bottom and left sides. if you do this over and over, will it ever get back to where it started? why or why not? if so, how many times will you need to do this until it is back where it started?



Back to Platonic Solids



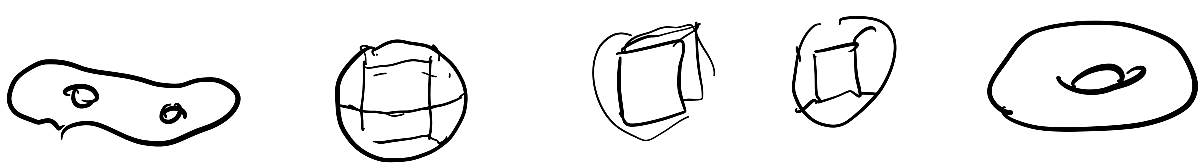
Get a planar graph
w/ # regions =
faces

Properties of graph:

- each face has same # of edges
- each vertex same degree (same # edges)

Euler's formula:

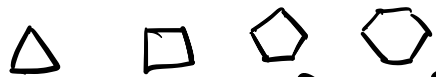
$$\boxed{(\#\text{faces}) - (\#\text{edges}) + (\#\text{vertices}) = 2}$$



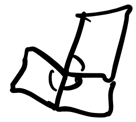
Euler Characteristic & Gauss-Bonnet

Last time: n = # vertices e = # edges f = # faces

k = # sides per face



k-gons



$180 - \frac{360}{6} = 180 - 60 = 120$

$k = 3$

$\frac{360}{k} = 120$

$180 - 120 = 60$

90

$k\varphi = 360$

$\varphi = \frac{360}{k}$

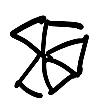
$$\boxed{\theta = 180 - \frac{360}{k}}$$

$180 - \frac{360}{5} = 180 - 72 = 108$

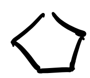
$\Delta \to 60$

\# faces to a corner

3	4	5
180	240	300



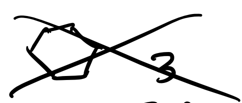
~~6~~
~~360~~



\# faces per corner

3 ~~4~~

324

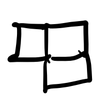


~~3~~
~~360~~

$\square \to 90$ # faces to a corner

3

270

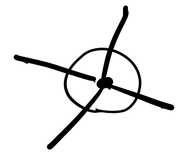


~~4~~
~~360~~

$2\#\text{edges} = fk$ $\qquad$ $\boxed{2e = fk}$

$f = \#\text{faces}$ $\qquad$ $k = \#\text{sides}$

$n = \#\text{vertices}$ $\qquad$ $f \cdot k = vd$



$\#$ of faces per vertex = degree of vertex $= d$

$\boxed{2e = fk \qquad fk = nd}$ $\qquad$ $\boxed{f - e + n = 2}$

$k = \#\text{sides}$ can be $3, 4, 5$

$d =$ $\quad 3,4,5 \quad 3 \quad 3$

$n = \#\text{vertices}$
$e = \#\text{edges}$
$f = \#\text{faces}$
$d = \#$ faces at each vertex
$k = \#$ sides for a face.

Cases:

$k=3, d=3$: $\boxed{2e = f3}$ $\qquad$ $3f = 3n$, $\boxed{f = n}$

$f - e + f = 2$

$2f - e = 2$

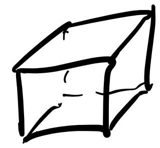
$e = 2f - 2$

$2(2f-2) = 3f$

$4f - 4 = 3f$

$k=3, d=4 \rightsquigarrow f = 8$

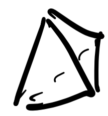
$k=3, d=5 \rightsquigarrow n = 6$



$f = 6 \quad e = 12$

$n = 8 \quad d = 3$

$k = 4$



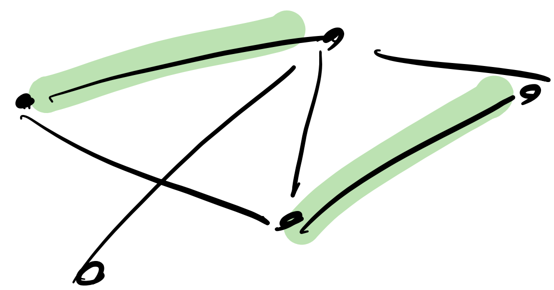
$f = 4$

$n = 4$

$e = 8 - 2 = 6$

Matching

a matching on a graph is a collection of edges, which have no common vertex

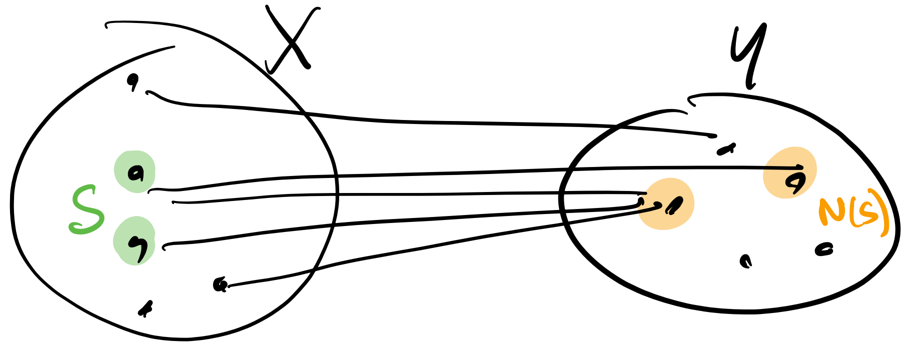


<u>Practice:</u> Bipartite graph

<u>Def</u> A bipartite graph is a graph G whose vertices have been partitioned into two disjoint subsets

$V(G) = X \cup Y$ where no two vertices in X are adjacent

& no two vertices in Y are adjacent



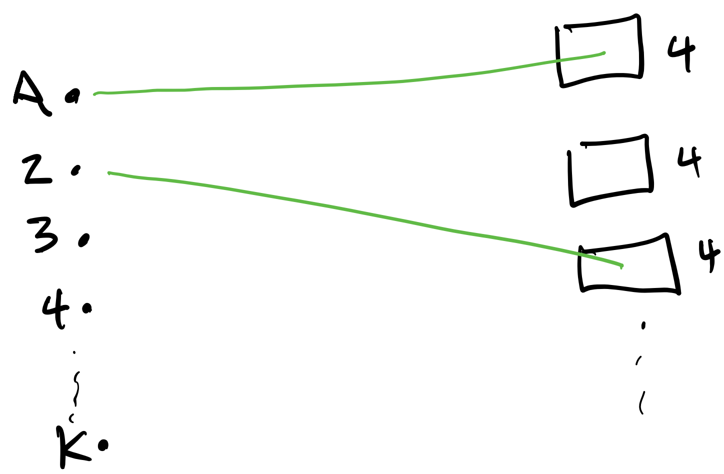
Hall's theorem

there is a matching which covers each vertex in X if and only if for each $S \subset X$, $\#N(S) \geq \#S$

Notation

if $S \subset X$ write $N(S) = \{$vertices adjacent to some vertex in $S\}$

ex:



X = values Y = piles

$S \subset X$ some collection of values

$N(S)$ = piles containing some of these values

Want to say $\#N(S) \geq \#S$

#of piles w/ those $\geq$ #values in S
values
in S