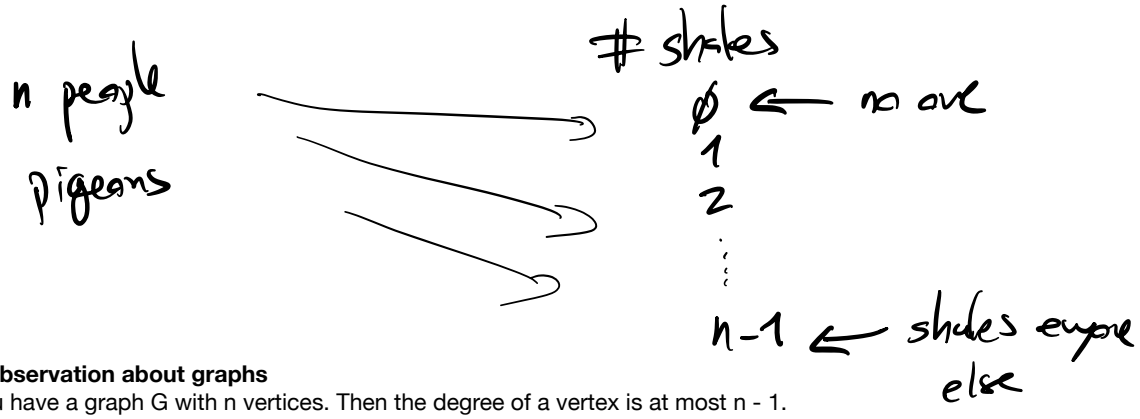
Puzzler:

useful fact (the pigeonhole principle)
Suppose you have n pigeons flying into k holes with k < n. Then there is some hole with at least 2 pigeons.

challenge
Show that if you have a party and some people shake hands, then there are always at least 2 people who have shaken hands with the same number of people.



Hint

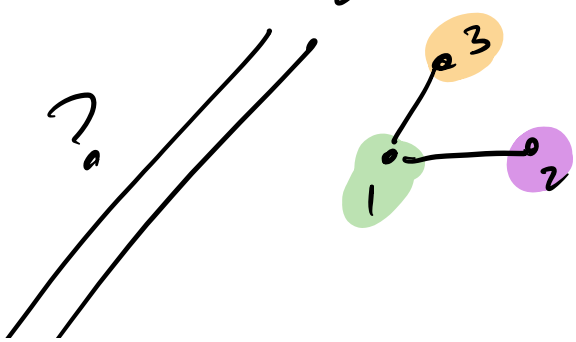
random(?) observation about graphs
Suppose you have a graph G with n vertices. Then the degree of a vertex is at most n - 1.

- Landscape of graphs
- Back to Hall's Marriage theorem
- Last application of graph theory: coloring

"Sameness"

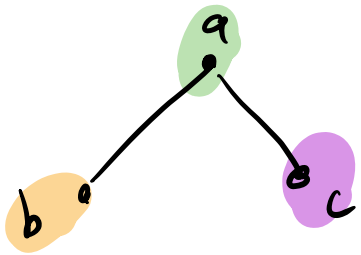
V = "{1,2,3}"

E = {{1,2}, {1,3}}



$\{a, b, c\}$ $\quad\quad$ $\{\{a, b\}, \{a, c\}\}$

V



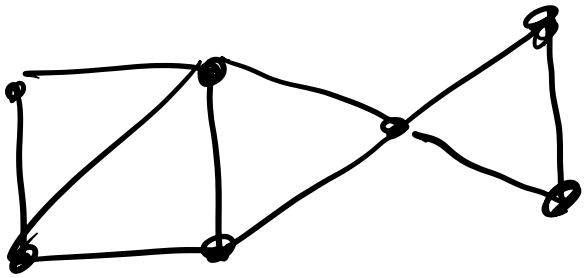
Definition An isomorphism between two graphs G, G' is a 1-1 correspondence between the vertices of G & G' which preserves adjacency.

$1 \longleftrightarrow a$

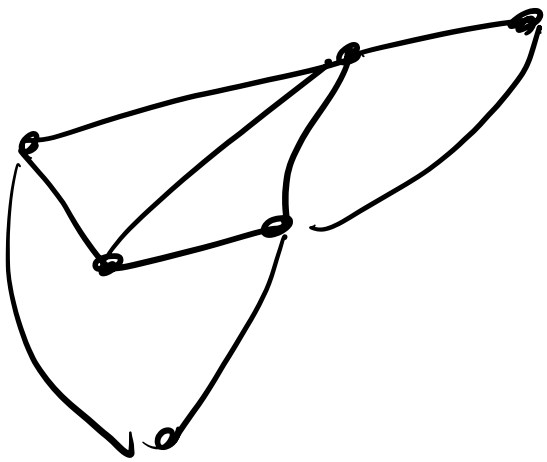
$2 \longleftrightarrow b$

$3 \longleftrightarrow c$

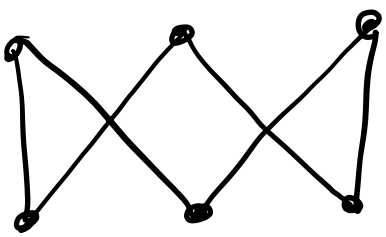
If there is an isomorphism between G & G' we say they are isomorphic.

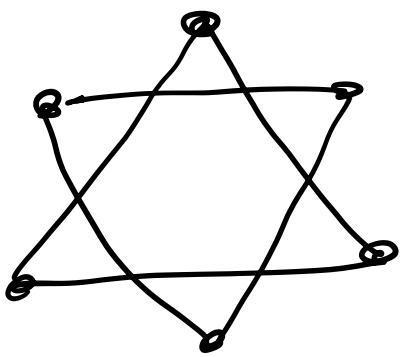


G



G'





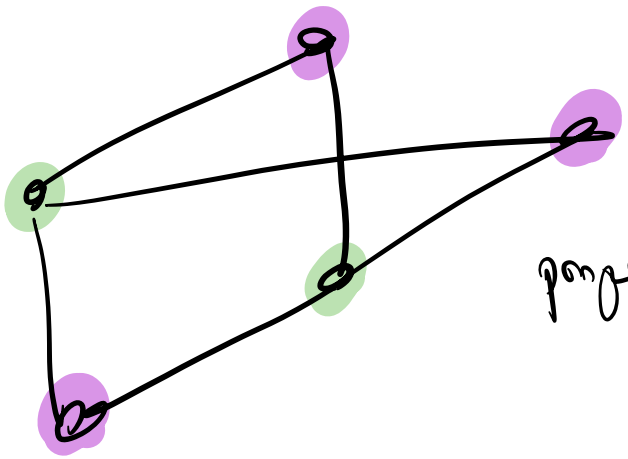
#vertices	#graphs
0	1
1	1
2	2
3	4
4	11
5	34
6	156
7	1,044
8	12,346
9	274,668
10	12,005,168
16	# digits

16 : 1,787,577,725,145,611,700,547,848,190,848

(31 digits)

Colorability

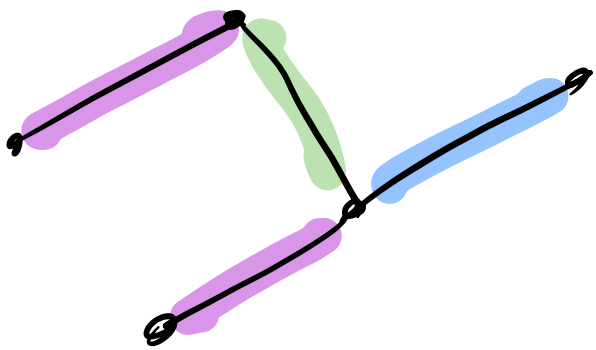
Definition: a (vertex) coloring of a graph is an assignment of a color to each vertex. It is called proper if adjacent vertices have different colors.



proper "2-coloring"

Edge colorings

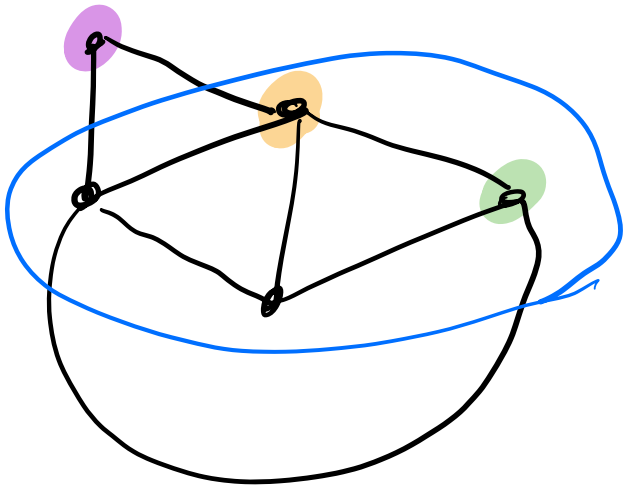
Def An edge coloring is an assignment of a color to each edge. It is called proper if adjacent edges have different colors.



G graph $\Delta(G) = \max$ deg of a vertex.

$\Delta(G) \leq$ # colors in an edge coloring $\leq \Delta(G)+1$

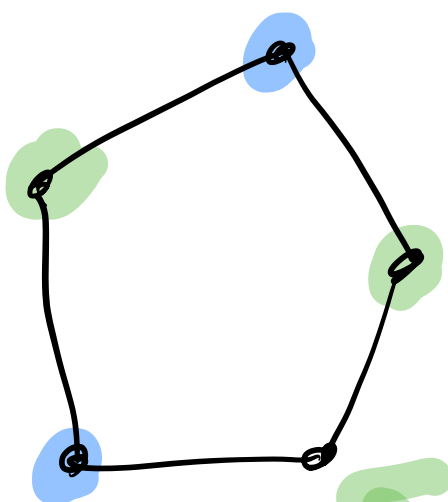
Vizys's theorem

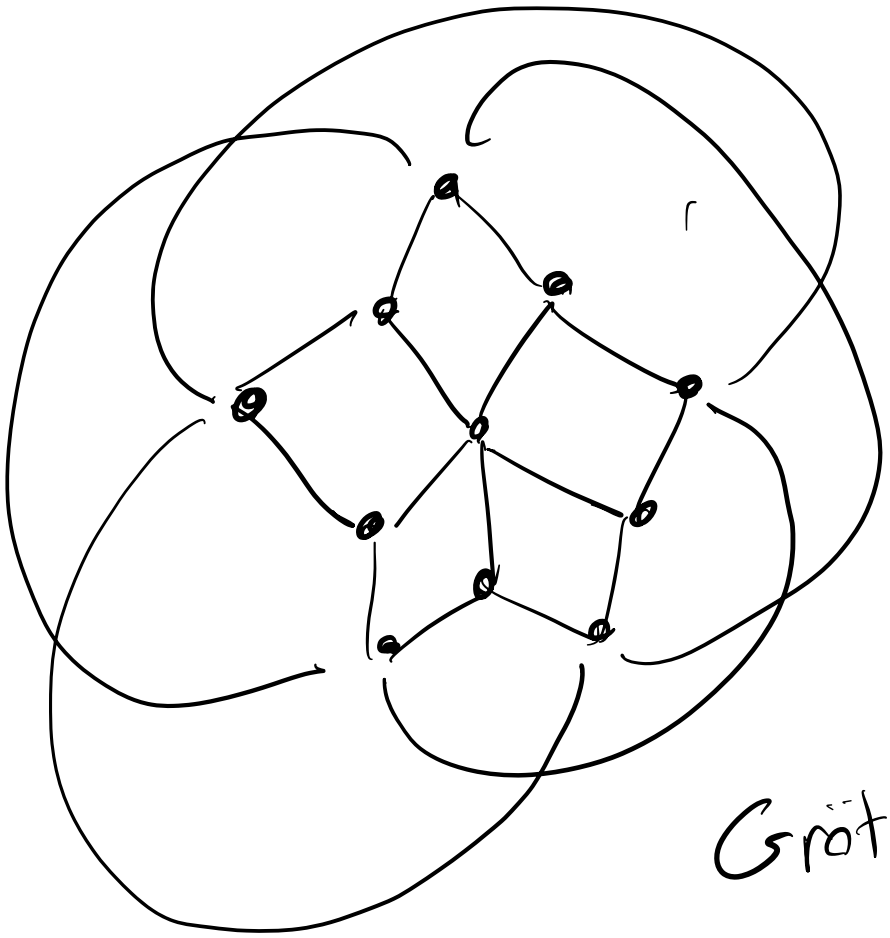


← group of vertices all mutually connected

4-"clique"

Cliques give lower bounds for #colors





need 4 colors

Grötzsch graph.

$(x-3)(x-2)(x-1)x(x^7-14x^6+\cdots-400x^4-\cdots)$

$4!$ $\quad x=4 \quad$ 520