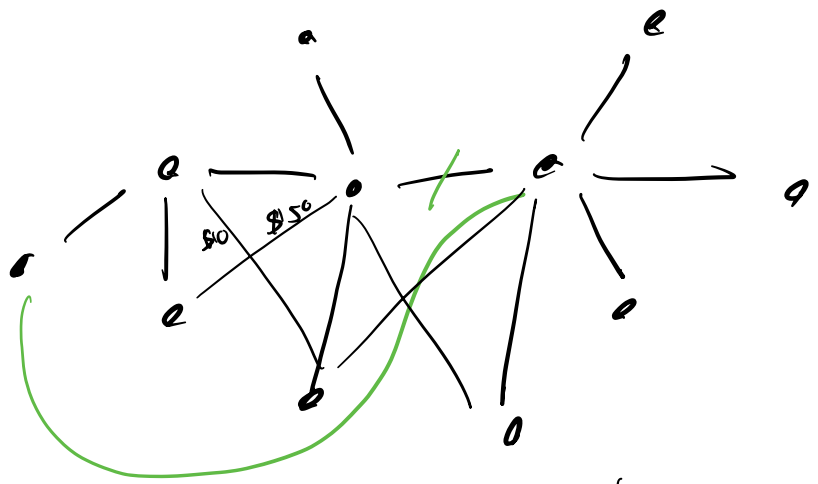
Puzzler If you list the first natural numbers up to 1,000,000 in alphabetical order (ignoring spaces) what is the first odd number in the list?

Herb Wilf
UPenn math
1962-2006
Author of
"generatingfunctionology"



optimize : minimize cost of connecting.
"minimall spanning tree"

cost ⟷ distance ?

minimize travel time. (w/ population in mind?)

The chromatic polynomial

$G \rightsquigarrow \chi_G(x)$

$G = \bullet \qquad \chi_\bullet(x) = x$

↗ #ways to color w/ 5 colors = 5

#color G = #{color G | x & y have same color} = #ways to color $\dot{G}$

+ #{color G | x & y have diff. colors} = #ways to color $\tilde{G}$

$$\chi_G(x) = \chi_{\tilde{G}}(x) + \chi_{\dot{G}}(x)$$

(#ways to color [n disconnected vertices] w/ x colors

x^n

$$\chi_{\tilde{G}}(x) = \chi_G(x) - \chi_{\dot{G}}(x)$$

$$\chi_{\text{path}}(x) = \chi_{\text{one edge + isolated vertex}}(x) - \chi_{\text{one edge}}(x)$$

$$\chi_{\text{one edge + isolated vertex}}(x) = \underbrace{\chi_{\text{three isolated vertices}}(x)}_{x^3} - \underbrace{\chi_{\text{two isolated vertices}}(x)}_{x^2}$$

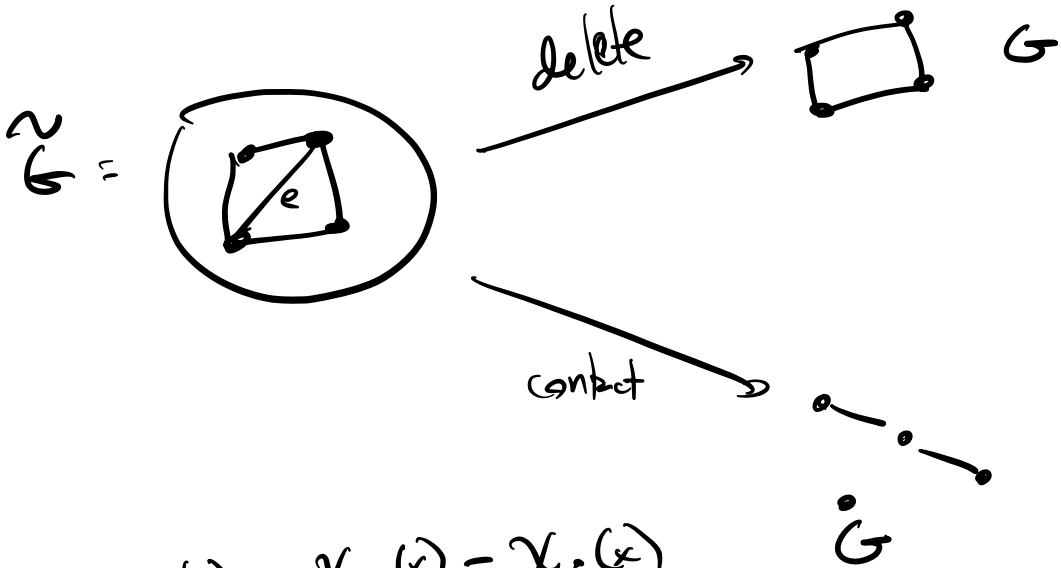
$$\chi_{\text{one edge}}(x) = \underbrace{\chi_{\text{two isolated vertices}}(x)}_{x^2} - \underbrace{\chi_{\text{one vertex}}(x)}_{x}$$

$$\chi_{\text{path}}(x) = (x^3 - x^2) - (x^2 - x)$$

$$= x^3 - x^2 - x^2 + x$$

$$= x^3 - 2x^2 + x$$

$$\chi_{(\square)}(2) = 2^3 - 2(2)^2 + 2 = 8 - 2\cdot 4 + 2 = 2$$



$$\chi_{\tilde{G}}(x) = \chi_G(x) - \chi_{\cdot G}(x)$$