

Puzzler: (A mathematical card trick - no sleight of hand)

Alice & Bob are demonstrating a magic trick to Carl

Carl chooses 6 cards and gives them to Alice

Alice chooses 1 of these and gives one back to Carl (his special card). Bob can't see any of these.

Alice arranges the 5 remaining cards in a pile and gives them to Bob. Bob examines the pile, thinks a bit and then tells Carl what his special card is. How?

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Today: Numbers (Chapter 1)

Extra detail: "Numerical notation, a comprehensive history" by Stephen Chrisomalis.

What are numbers?

How are they represented?

(Symbolic as opposed to verbal)

683592

Symbolic reps of numbers  
are "technology"

- not universal (nor is unity)
- subject to lots of transference, evolution, borrowing
- independent developments
- lots of variation (w/ lots of common themes)

12	western numerals	Hindu-Arabic		
𐎠𐎢𐎡𐎠	Hittite Hieroglyphs	(~1500-1200 BCE)		
⋮	Mayan			
𐎠𐎡𐎠	Babylonian cuneiform			
XII	Roman numerals	↪	22	XXII
110	Egyptian			
12	Arabic			
十二	Traditional Chinese	↪	<del>十二</del>	22
			= 十 =	

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Modern systems (from CS)

12:	1100	Binary
	14	Octal
	B	Hexadecimal

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What is a number?

3 is many representing threeness.

Quantity  $\longleftrightarrow$  number

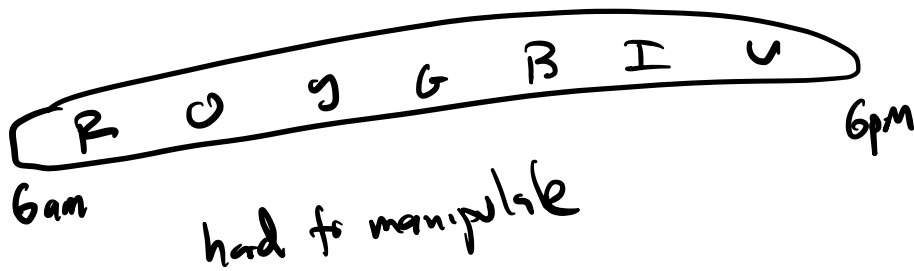
3 o'clock

"interval of the past  
noon"

3 apples

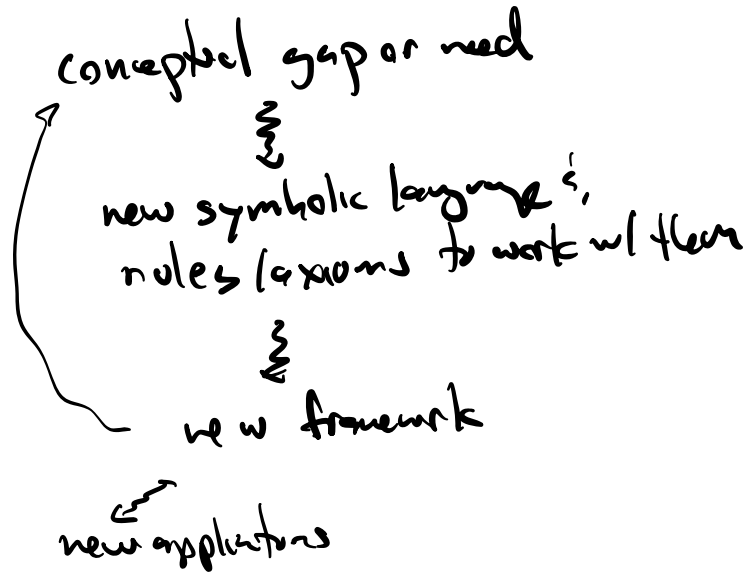
Why does it make sense to use some language  
for quantities of apples as intervals of time?

Why not use color (spectrum) to represent time?



(XVII) · (CXII)

# Typical math development



Gravity

how do thys fall?

→ they if gravity

→ gives applications to planetary orbits.

Numbers

need to tally what's been

→ number framework!  
symbolic rep.

dates!  
calendars

→  
ordering  
thys

→ measure

Def Numerical thys are thys that can be measured about usg numbers.

Def Numbers are thys that count.

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Let's count!

Mayan

$$\textcircled{1} = 0$$

$$\cdot = 1$$

$$\cdot\cdot = 2$$

$$\cdot\cdot\cdot\cdot = 4$$

$$- = 5$$

$$\cdot\cdot\cdot = 8$$

$$\overset{\cdot}{=} = 11$$

$$\overset{\cdot\cdot}{=} = 18$$

$$\overset{\cdot\cdot\cdot}{=} = 19$$

$$\overset{\cdot}{\textcircled{1}} = 20$$

$$\overset{\cdot}{\cdot} = 21$$

$$\overset{\cdot}{\cdot\cdot} = 22$$

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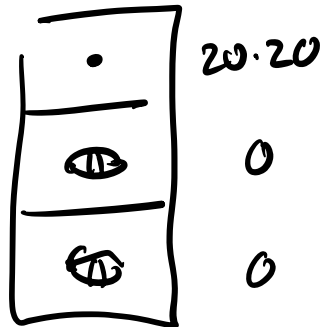
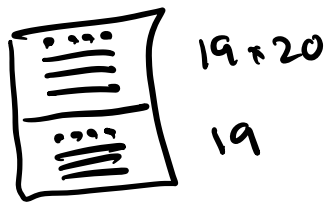
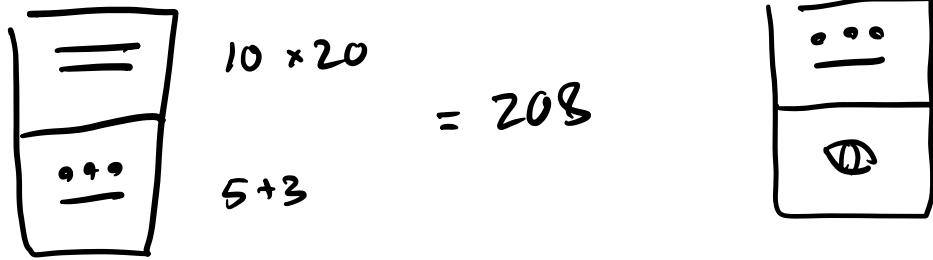
$$\overset{\cdot}{\cdot\cdot\cdot} = 37$$

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$$\overset{\cdot\cdot}{\textcircled{1}} = 40$$

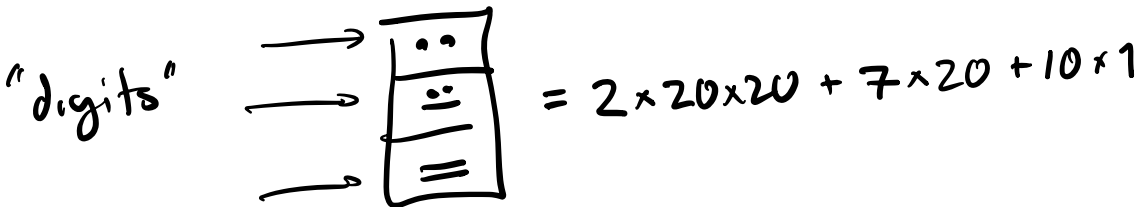
$$\overset{\cdot\cdot}{\cdot\cdot} = 42$$

$$\overset{\cdot\cdot}{\cdot\cdot\cdot} \begin{matrix} - 10^0 \\ \setminus 3 \end{matrix} = 103$$



$$537 = 5 \times 10 \times 10 + 3 \times 10 + 7 \times 1$$

↑    ↑    ↑  
 100's 10's 1's  
 10 · 10



# Babylonian cuneiform

$$\top = 1$$

$$\top\top\top = 3$$

$$\top\top\top = 4$$

$$\begin{array}{c} \top\top\top \\ \top\top\top \\ \top\top\top \end{array} = 9$$

$$\top\top\top = 5$$

$$\top = 10$$

$$\top\top\top = 30$$

$$\top\top\top \top\top\top = 38$$

$$\top\top\top \top\top = 34$$

$$\begin{array}{c} \top\top\top \\ \top\top\top \end{array} \begin{array}{c} \top \\ \top \end{array} = 59$$

$$\begin{array}{c} \top \\ \uparrow \\ 1 \cdot 60 \end{array} \quad \begin{array}{c} \top\top \\ \uparrow \\ 10 \cdot 1 \end{array} = 71$$

$$\begin{array}{c} \top\top \\ 20 \cdot 60 \end{array}$$

$$\begin{array}{c} \top\top\top \\ 22 \end{array}$$

$$= 1222$$

$$\top\top\top \quad \top\top\top = ?$$

# Positional # systems

fixed set of "digits" "signs" "glyphs" "symbols"  
value depends on placement

↙ corresponds to "powers of the base"

Base	1	10	$10 \times 10$ $10^2$	$10 \times 10 \times 10$ $10^3$	
10					
20			$20 \times 20$	...	
60			$60 \times 60$	...	

Sign Value: different signs for diff. values.  
com. sign value.

X            C            MCC  
                                  IV