Puzzler:

What's the next term in this sequence?

1, 11, 31, 211311, 131112211321, ...

122113311331222113121 7

Hint:

what's the next term in this sequence?

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

Gradient flows

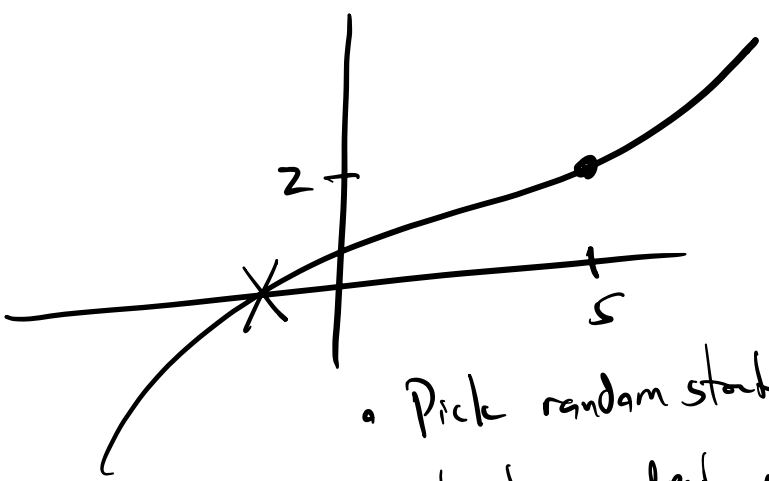
have some eqn trying to solve $f(x) = 0$

$x^2 = 9$ ↓ $x = 3$ $\quad x^2 = 10$ $x = \sqrt{10}$

$e^x + \sin x = 7$

$x - \cos x = \log x$ $\quad e^x + x = 5$ $\quad x = W(5)$

$e^x = 5$ ↓ $x = \ln 5$



$f(x) = 0$?

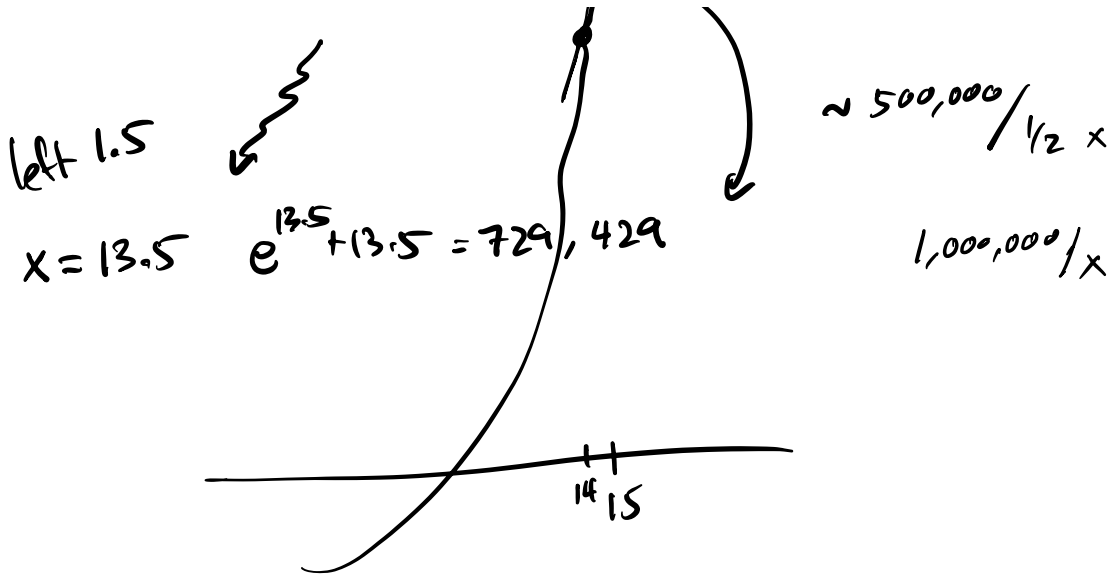
$x = 5 \rightsquigarrow f(5)$

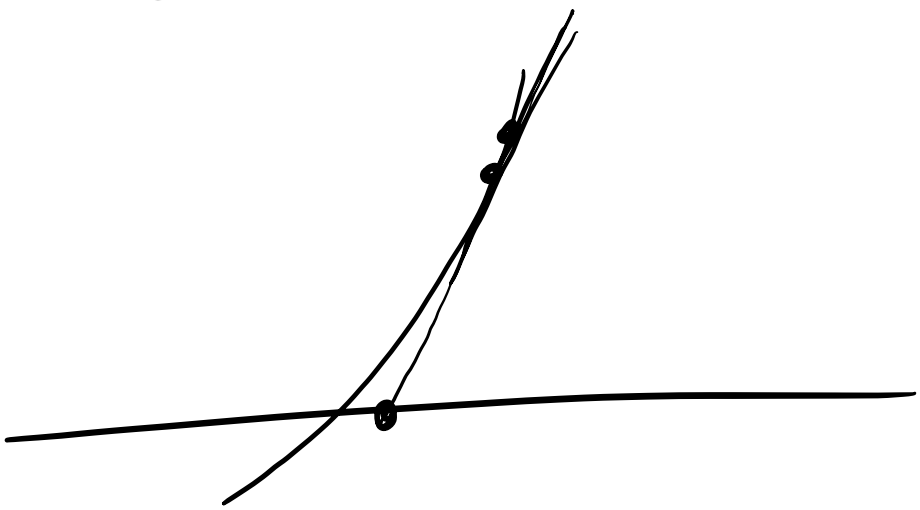
- Pick random starting point.
- check gradient nearby (slope)
- adjust your guess based on this

$e^x + x = 17$ $\quad e^x + x - 17 = 0$

guess $x = 15$ $\quad e^{15} + 15 = 3{,}269{,}032$

$e^{14} + 14 = 1{,}2029\ldots$ ~ 200,000 ×'s / 1 ×





Square root

$x^2 = 10$ $\quad x = 3$ $\quad 3^2 = 9$

$(3+\varepsilon)^2 = 9 + 3\varepsilon + 3\varepsilon + \varepsilon^2 = 9 + 6\varepsilon$

ε = very small number.

ε^2 = too small to matter.

near $x = 3$

change x to $x+\varepsilon$

value changes

from 9 to $9+6\varepsilon$

$9+6\varepsilon = 10 \qquad \varepsilon = 1/6$

$6\varepsilon = 1$

$\left(3+\frac{1}{6}\right)^2 = 9+1+\frac{1}{36}$

$\approx \boxed{10+\frac{1}{36}}$

$= \frac{360}{36} + \frac{1}{36} = \frac{361}{36}$

$\boxed{3+\frac{1}{6}}$ 1st correction.

$\frac{18}{6} + \frac{1}{6} = \boxed{\frac{19}{6}}$

$$\left(\frac{19}{6}+\varepsilon\right)^2 = \left(\frac{19}{6}\right)^2 + 2\left(\frac{19}{6}\right)\varepsilon + \varepsilon^2$$

$$= \frac{361}{36} + \frac{19}{3}\varepsilon \qquad \left(= \frac{360}{36}\right)$$

$$= \frac{361}{36} + \frac{12\cdot 19}{36}\varepsilon = \frac{360}{36}$$

$$361 + 12\cdot 19\varepsilon = 360$$

$$12\cdot 19\varepsilon = -1$$

$$\varepsilon = -\frac{1}{12\cdot 19}$$

new answer:

$$\sqrt{10} \approx \frac{19}{6} - \frac{1}{12\cdot 19} = \frac{19}{6} - \frac{1}{228}$$

$= 3.1622807$

$\sqrt{10} = 3.162277$

Newton's method

Old school derivatives:

$f(x)$ $\quad x = a$ want to know slope near $x = a$

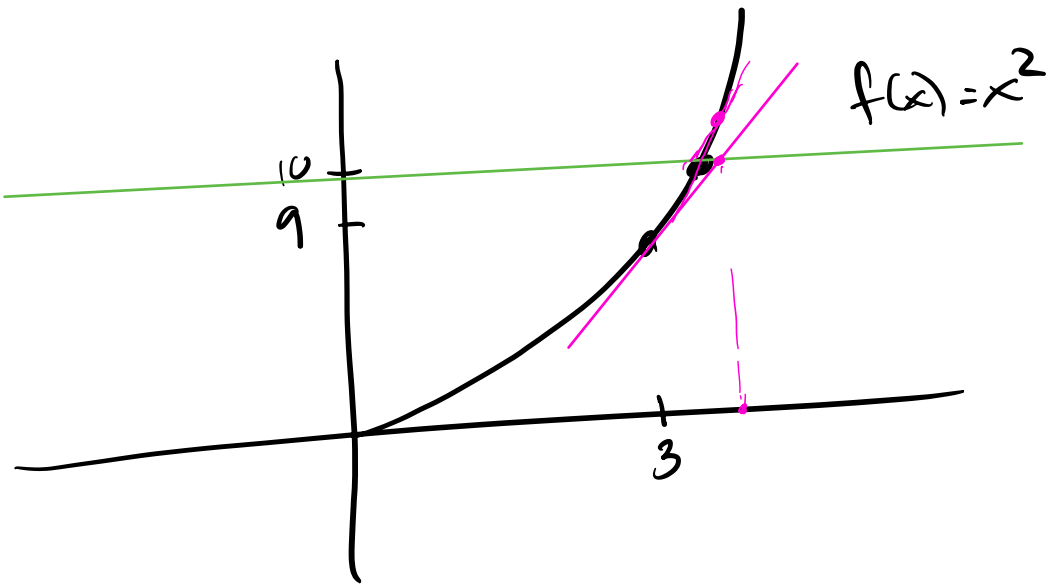
$$f'(a) = \frac{f(a+\varepsilon) - f(a)}{\varepsilon}$$ ε "small"

ignore factors of ε^2 ...

$f(x) = x^2$

$$\frac{f(a+\varepsilon) - f(a)}{\varepsilon} = \frac{(a+\varepsilon)^2 - a^2}{\varepsilon} = \frac{a^2 + 2a\varepsilon + \varepsilon^2 - a^2}{\varepsilon}$$

$$= \frac{2a\varepsilon}{\varepsilon} = \boxed{2a}$$



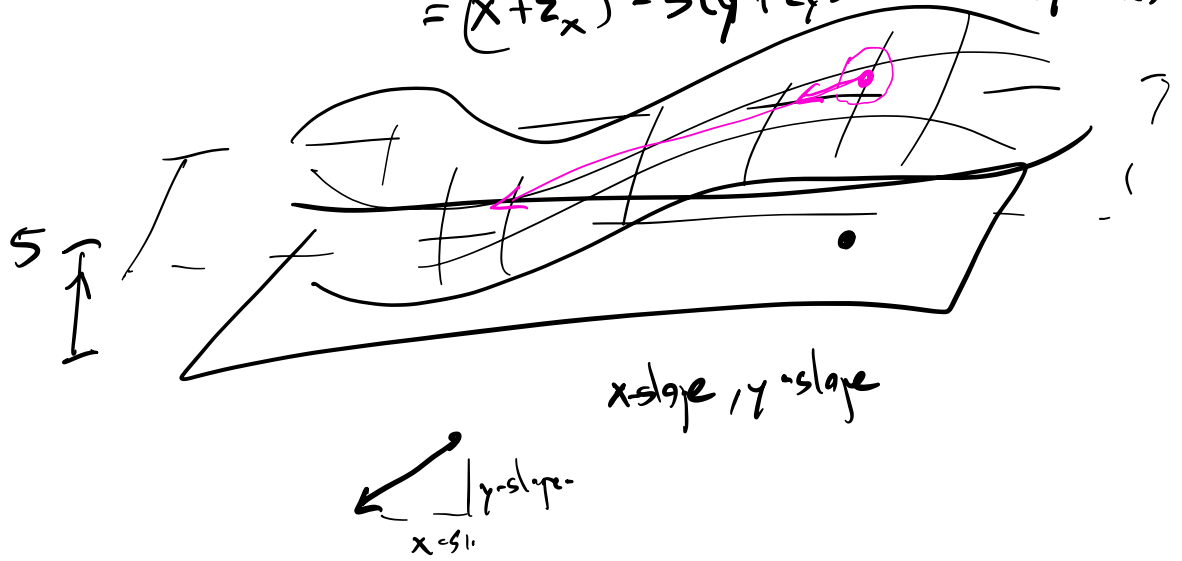
$f(x) = 5$

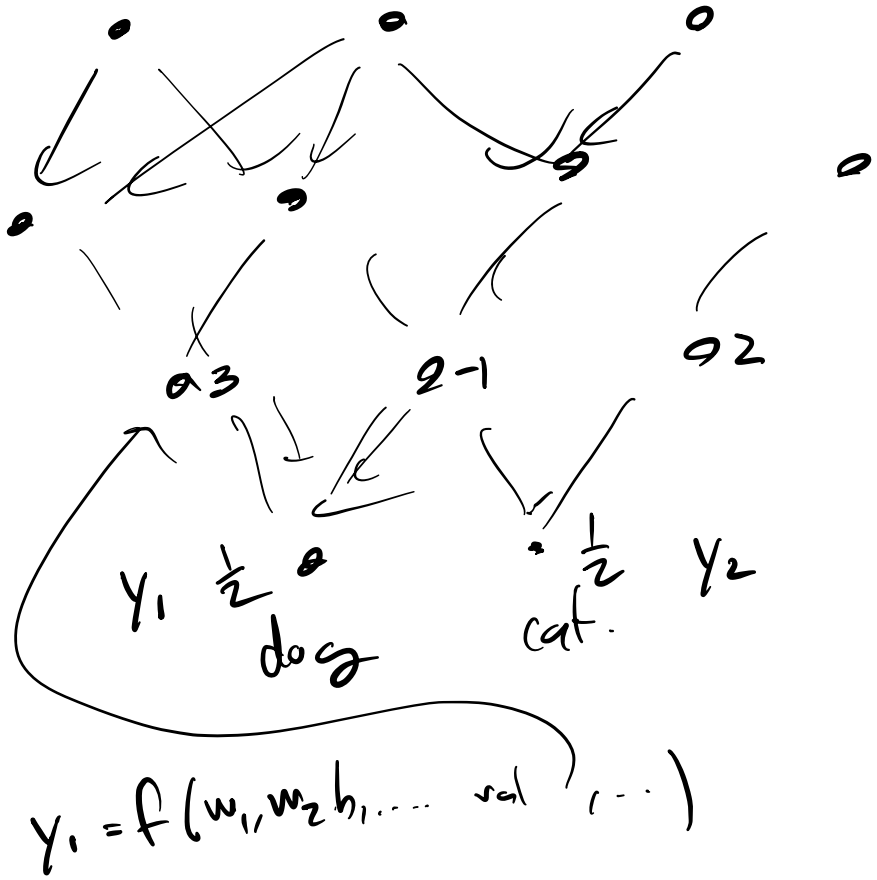
$f(x_1, x_2, x_3 \ldots, x_{17}) = 5$

$f(x, y) = 5$

$f(x, y) = x^3 - 3y^2 + xy = 5$

$= (x + \varepsilon_x)^3 - 3(y + \varepsilon_y)^2 + (x + \varepsilon_x)(y + \varepsilon_x) = 5$





$y_1 = f(w_1, w_2 h_1 \ldots \text{val} \ldots)$