Puzzler:

If mix is 1000110 and mimic is 10001100011100
what is civil?

Today: Sets

From 300BCE to ~ 1700's CE
dominant influence & "foundations" of math
was Euclid's Elements of Geometry.

Explosion of new ideas 1700's & 1800's
Clear that math needed "new foundations"
Georg Cantor's theory of sets 1870's.

Intuitively, a set is an unordered collection of objects (called elements or members)

examples: $\{1,2,3\}$ the set whose elements are 1, 2, & 3

$S = \{R, G, B\}$

$H = \{1, 2, 3, 4, \ldots, 100\}$ ellipses to prevent boredom.

$U = \{\text{Alabama}, \text{Alaska}, \ldots, \text{Wyoming}\}$

Useful notation:

"Set builder notation"

$X = \{2x+1 \mid x = 0, 1, 2, \ldots, 100\}$

equiv. to $X = \{1, 3, 5, 7, \ldots, 201\}$

$X = \{n \mid n \text{ is an odd whole number between } 1 \text{ \& } 201\}$

$\in$ $\in$

$S = \{1, \ldots, 100\}$ we write $x \in S$ to mean x is an element of S

$\{3x-1 \mid x \in S\}$

$\{3x-1 \mid x \text{ is an element of } S\}$

$\{\ \}$ 

Note: repetition doesn't mean anything

$\{1,1,2,2\} = \{1,2\}$

Union: If X, Y are sets

then $X \cup Y$ is the set whose elements are those in X together with those in Y.

$$X \cup Y = \{x \mid x \in X \text{ or } x \in Y\}$$

ex: $X = \{1,2,3\}$ $Y = \{2,4,6,8,10\}$

$$X \cup Y = \{1,2,3,4,6,8,10\}$$

$$X \cup X = \{1,2,3\} = X$$

$$X \cap Y = \{x \mid x \in X \text{ and } x \in Y\}$$

$$X \cap Y = \{2\}$$

$\emptyset$ = the empty set

$$\emptyset = \{\}$$

Subsets

we say A is a subset of B and write $A \subseteq B$ if every element of A is also an element of B

ex: $\{1,2\} \subseteq \{1,2,3\}$ $\quad \emptyset \subseteq \{1,2\}$ yes!

if $A \subseteq B$, we write $B \setminus A$ (complement of A in B) to mean elements of B not in A

ex: $\{1,2,3\} \setminus \{1,2\} = \{3\}$

Tongue twisters to get used to the notation

$\{1,2,3\}$ 3

$\{1,2,3,3\}$ 3

$\{1,2,\{1,2\},3\}$ 4 elements: 1, 2, 3, & the set $\{1,2\}$

$\{1,\{1\},\{1,\{1\}\}\}$ 3 elements

How many elements?

1. $\{\emptyset\}$ 1

2. $\{\{\emptyset\},\emptyset\}$ 2

3. $\{\{1,2\},\emptyset,\{1\},\emptyset\}$ 3

$\mathbb{N} = \{0,1,2,3,\dots\}$ this is an infinite set.

Cantor was heavily (famously) criticized for using the concept of infinite sets.

Debate on origins of Calculus (Newton & Leibniz)

Story: Newton won because Leibniz didn't make sense.

Leibniz used notation of "infinitesimals"

$\frac{dy}{dx}$

until 1960's Abraham Robinson realized infinitesimals could be made precise.

Let $X = \{n \mid n$ a natural number which can be described by a sentence with less than 100 words$\}$

let m = smallest number which can't be described by a sentence with less than 100 words.