

$X \rightarrow \text{uniform on } [a, b]$

$$f(x) = \begin{cases} C & x \in [a, b] \\ 0 & \text{else} \end{cases}$$

$$\int f(x) dx = \int_a^b C dx = 1$$

$$= \begin{cases} \frac{1}{b-a} & x \in [a, b] \\ 0 & \text{else} \end{cases}$$

Gamma Random variable

exp: p.d.f.
 $f(x) = \lambda e^{-\lambda x}$

$$f(x) = C x^\alpha e^{-bx}$$

$$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$$

$\alpha = 1 \leftrightarrow \text{exponential}$
 $(\beta = \lambda^{-1})$

Gamma function

\bullet $\alpha = 1 \quad \beta = \lambda^{-1} \quad \text{exponential}$
 $2\alpha = \gamma \quad \beta = 2 \quad \text{"Chi-square distribution with } \gamma \text{ degrees of freedom"}$
 $\uparrow \text{integer } \geq 1$

$$M_X(t) = (1 - \beta t)^{-\alpha}$$

Given independent, exponential vars X_1, \dots, X_n , $f(x) = \lambda e^{-\lambda x}$

$$M_{X_1 + \dots + X_n}(t) = \prod M_{X_i}(t)$$

$$= \prod_{i=1}^n (1 - \lambda^{-1} t)^{-1}$$

$$= (1 - \lambda^{-1} t)^{-n}$$

$\sum_{i=1}^n X_i$ is a gamma variable w/ $\alpha = n$
 $\beta = \lambda^{-1}$

Normal random variables

X a normal random var \leftrightarrow p.d.f.

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma} \right)^2}$$

variance σ^2
mean μ

$$M_X(t) = e^{\mu t + \frac{1}{2} \sigma^2 t^2}$$

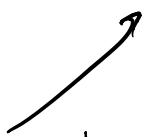
$$M_{X_1+X_2}(t) = M_{X_1}(t) M_{X_2}(t)$$

X_1 normal σ_1^2, μ_1

$$= e^{\mu_1 t + \frac{1}{2} \sigma_1^2 t^2} e^{\mu_2 t + \frac{1}{2} \sigma_2^2 t^2}$$

X_2 --- σ_2^2, μ_2

$$= e^{(\mu_1 + \mu_2)t + \frac{1}{2} (\sigma_1^2 + \sigma_2^2)t^2}$$



$X_1 + X_2$ is normal w/ mean $\mu_1 + \mu_2$
and variance $\sigma_1^2 + \sigma_2^2$

$X_1, X_2, X_3, \dots, X_n, \dots$ identical r.v.s \leftarrow mean μ
 variance σ^2

$$\mu\left(\sum_{i=1}^n X_i\right) = \sum_{i=1}^n \mu(X_i)$$

$$E[\sum X_i] = \sum E[X_i]$$

$$\text{Var}\left(\sum_{i=1}^n X_i\right) = \sum_{i=1}^n \text{Var} X_i$$

$$\mu(\sum X_i)$$

$$\mu(\lambda X) = \lambda \mu(X)$$

$$\text{Var}(\lambda X) = \lambda^2 \text{Var} X$$

$$X = \sum_{i=1}^n X_i \quad \mu(X) = n\mu \quad \sigma_X^2 = \text{Var} X = n\sigma^2$$

$$\frac{X - n\mu}{\sqrt{n}\sigma} \quad \leftarrow \begin{array}{l} \text{random var w/ mean 0} \\ \text{variance 1.} \end{array}$$

$$\underline{\text{Central limit:}} \quad \text{c.d.f. of } \frac{\left(\sum_{i=1}^n X_i\right) - n\mu}{\sqrt{n}\sigma}$$

approaches c.d.f. of Z as $n \rightarrow \infty$

Example

Catch fish in river mean 5.3 lbs
std dev 1.2 Variance ~ 1.4

After catch 10 fish, what's the prob. that weight ≥ 50 lbs
of fish?

$$X = X_1 + \dots + X_{10}$$

$$X \geq 50$$

$$\frac{X - 10 \mu}{\sqrt{n} \sigma} \approx Z$$

$$X - 53 \geq 50 - 53$$

$$Z \approx \frac{X - 53}{\sqrt{10}(1.2)} \geq \frac{-3}{\sqrt{10}(1.2)} \approx -0.8$$

$$P(Z \geq -0.8) \approx 89\%$$

