

$$My|x = \alpha + \beta x$$

Part 4

Normal Regression analysis

Additional assumption: bivariate x, and distribution

$f(y|x)$ normal w/ mean $\alpha + \beta x$
w/ variance σ^2 not depnd on x

$$f(y|x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(y - (\alpha + \beta x))^2}$$

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$$f(y|x; \alpha, \beta, \sigma^2)$$

get MLE statistics for α, β, σ^2

choose (bivariate \vec{x}, \vec{y} $x_1, y_1, x_2, y_2, \dots$)

want to maximize likelihood fun.

$$L(\alpha, \beta, \sigma^2; \vec{x}, \vec{y}) = \prod f(y_i|x_i; \alpha, \beta, \sigma^2)$$

(in practice max $\ln L$)

$$\bar{x} = \frac{1}{n} \sum x_i$$

get estimates $\hat{\alpha}, \hat{\beta}, \hat{\sigma}^2$

$$\hat{\beta} = \frac{S_{xy}}{S_{xx}}$$

$$S_{xx} = \frac{1}{n} \sum (x_i - \bar{x})^2$$

$$S_{xy} = \frac{1}{n} \sum (x_i - \bar{x})(y_i - \bar{y})$$

$$\hat{\alpha} = \bar{y} - \frac{S_{xy}}{S_{xx}} \bar{x}$$

$$\hat{\sigma}^2 = \sum_{i=1}^n (y_i - (\hat{\alpha} + \hat{\beta} x_i))^2$$

Regression analysis perspective

x_i 's are fixed (not random vars)
but y_i 's are

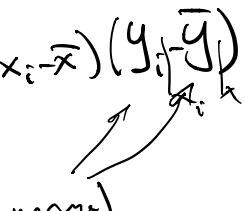
Ex Plants, examine ht after 1 day, 7 days,
14 days.

$$\begin{array}{ll} x_1 = 1 & y_1 = \text{ht at day 1} \\ x_2 = 7 & y_2 = \dots 7 \\ x_3 = 14 & y_3 = \dots 14 \end{array}$$

from this perspective
sample test. for $\hat{\beta} = \frac{S_{xy}}{S_{xx}}$

$$= \frac{1}{S_{xx}} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$$

$\hat{\beta}$ is a normal random variable



$$\mu_{\hat{\beta}} = \beta \quad \sigma_{\hat{\beta}}^2 = \frac{\sigma^2}{S_{xx}}$$

$$\hat{\sigma}^2 = \sum_{i=1}^n (y_i - (\hat{\alpha} + \hat{\beta}x_i))^2$$

turns out: $n \frac{\hat{\sigma}^2}{\sigma^2}$ is a χ^2_{n-2} dist. random var. and is indep. of $\hat{\beta}$.

consequently

$$\left(\frac{\hat{\beta} - \beta}{\sigma / \sqrt{S_{xx}}} \right) \sqrt{\frac{(n \hat{\sigma}^2) / \sigma^2}{n-2}}$$

is a t dist.
random var w/
 $n-2$ d.f.
of freedom

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$$- \left(\frac{\hat{\beta} - \beta}{\hat{\sigma}} \right) \sqrt{\frac{(n-2) S_{xx}}{n}}$$

const.

Part 5

Normal convolution analysis