The Plot

have some populations w/ some features (parameters)

e.g. mean μ, variance σ^2

take measurements $X_1, \ldots, X_n$ iid

Calculate sample mean $\bar{X} = \frac{\sum X_i}{n}$ ask: how well does $\bar{X}$ approximate μ?

Goal: want a statement:
we are 95% certain that μ is between $\bar{X}-3$ & $\bar{X}+3$

i.e. $P(\bar{X}-3 \le \mu \le \bar{X}+3) = .95$

given $\bar{x}$ measured value for $\bar{X}$ we're 95% certain that $\bar{x}-3 \le \mu \le \bar{x}+3$.

Def If we have some parameter θ for our population & random vars $\hat{\theta}_1, \hat{\theta}_2$ such that

$$P(\hat{\theta}_1 < \theta < \hat{\theta}_2) = 1-\alpha$$

and if $\hat{\theta}_1, \hat{\theta}_2$ are measured values of $\hat{\theta}_1$ & $\hat{\theta}_2$ then we say $\hat{\theta}_1 \le \theta \le \hat{\theta}_2$ is a $1-\alpha$ confidence interval for θ. we say $\hat{\theta}_1, \hat{\theta}_2$ are the lower & upper confidence limits and $1-\alpha$ the degree of confidence.

Note: these are not unique.

Example

population mean μ (known σ^2)

$\bar{X}$ if n is large $\bar{X} \approx$ normal random var.

$$\left.\begin{aligned} \mu(\bar{X}) &= \mu \\ var(\bar{X}) &= \frac{1}{n}\sigma^2 \end{aligned}\right\} \quad \frac{\bar{X}-\mu}{(\sigma/\sqrt{n})} = Z$$

$$\text{stddev}(\bar{X}) = \frac{\sigma}{\sqrt{n}}$$