

## Matrix groups

Def A matrix group is a group  $G$  which can be realized as a subgroup of the group  $GL_n(F)$  for a field  $F$ , as the set of solutions to polynomial equations in the entries of the matrix w/cells in  $F$ .

Ex:  $SL_n$  det

$$O_n = \{ T \mid T^t T = I_n \}$$

$$\sum_{j=1}^n t_{ji} t_{jk} = \begin{cases} 1 & \text{if } i=k \\ 0 & \text{if } i \neq k \end{cases}$$

$$U(2) = \{ T \mid T^* T = I_n \}$$

not closed ... in  $GL_2(\mathbb{C})!$

actually  $GL_2(\mathbb{C}) \hookrightarrow GL_4(\mathbb{R})$   
and closed there.

$$\begin{pmatrix} a_0 + a_1 i & b_0 + b_1 i \\ \cdot & \cdot \end{pmatrix} \hookrightarrow \begin{pmatrix} a_0 & -a_1 & b_0 & -b_1 \\ a_1 & a_0 & -b_1 & b_0 \\ c_0 & \cdot & \cdot & \cdot \end{pmatrix}$$