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Question: what are we doing here?
what's the point of being in this class?

Main theme: Metric space — notion of distance/closeness

closely connected to property of real #s $\mathbb{R}$ (analysis/calculus)

grading policy:
see website.

Metric Spaces & limits (Libl 7.1, 7.3)

Definition If X is a set, a metric on X is a function $d : X \times X \longrightarrow \mathbb{R}$

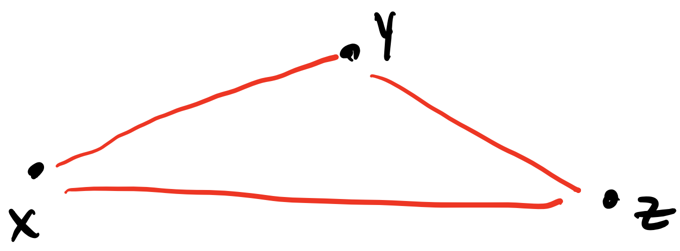
$$(x,y) \longmapsto d(x,y)$$

(1) $\forall x,y \in X$, $d(x,y) \geq 0$ (nonnegativity)

(2) if $d(x,y) = 0 \iff x = y$ (indiscernibles)

(3) $d(x,y) = d(y,x)$ (symmetry)

(4) $d(x,y) + d(y,z) \geqslant d(x,z)$



Examples

1) standard distance in line $\mathbb{R}$

$X = \mathbb{R}$ if $x, y \in \mathbb{R}$ $d(x,y) = |x-y|$

check: $d(x,y) = |x-y| \geqslant 0$ ✓ (1)

$d(x,y) = 0 \Rightarrow |x-y| = 0$ ✓

$\Rightarrow x - y = 0 \Rightarrow x = y$ (2)

$d(x,y) = |x-y| = |-(y-x)|$

$= |y-x| = d(y-x)$ (3) ✓

x, y, z $d(x,y) + d(y,z)$

$= |x-y| + |y-z|$

$d(x,z) = |x-z| = |x-y+y-z|$

$|A+B| \leq |A| + |B|$

if $A < 0$ $B > 0$ and $|A| \leq |B|$

then $|A+B|$ $|B| = B \geq -A = |A|$.

$B \geq -A \geq 0$

$0 \geq -A - B \geq -B$

$0 \leq A + B \leq B$

$|A+B| \leq |B| \leq |B| + |A|$

Slilur argument:

$|A+B|$

if A & B have diff signs can assume

$A > 0$ $B < 0$

consider $|A|$ $|B|$

either $|A| \geq |B|$ or $|B| \geq |A|$

Assume that $|A| \geq |B|$

then $\underset{\overset{\shortparallel}{A}}{|A|} \geq \underset{\overset{\shortparallel}{-B}}{|B|}$ $A \geq -B \geq 0$

$0 \geq -B - A \geq -A$

$0 \leq B + A \leq A$

$$0 \le |B+A| \le |A| \le |A| + |B|$$

$$A+B \le A$$

if $|B| \ge |A|$

$$|A+B| = |(-A) + (-B)|$$

$$(-A) < 0 \qquad (-B) > 0$$

$$|(-B)| \ge |A|$$

pos is bigger one, so by previous case

$$|A+B| = |(-A) + (-B)| \le |-A| + |-B|$$
$$= |A| + |B|$$

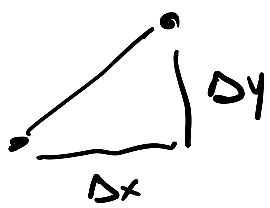
Ex: X any set

$$d(x,y) = \begin{cases} 1 & \text{if } x \ne y \\ 0 & \text{if } x = y \end{cases}$$

Example:

Distance in the plane $X = \mathbb{R}^2$

$$d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 - (y_1 - y_2)^2}$$



more generally: $X = \mathbb{R}^n$

$$d(x, y) = \sqrt{\sum (x_i - y_i)^2}$$

$$x = \vec{x} = (x_1, \ldots, x_n)$$

$$y = \vec{y}$$

To prove this is a metric uses the "Cauchy-Schwartz" Inequality.