modification of problem 1:

Show that for each n, $\exists \varepsilon > 0$ s.t. $B_\varepsilon(a_n)$ contains at most finitely many other points a_m for $m \in \mathbb{Z}_{\geq 0}$.

From last time:

Suppose X is compact, $x \in X$, $Y \neq \emptyset$ complete.

$f : X \setminus \{x\} \longrightarrow Y$ is continuous.

Then f is uniformly cont $\Longleftrightarrow \exists \tilde{f} : X \longrightarrow Y$ continuous such that $\tilde{f}(x') = f(x')$ for $x' \in X \setminus \{x\}$.

Pf: suppose $\exists \tilde{f}$ as above. WTS: f is unif. continuous.

Since X is compact, $\tilde{f}$ is unif. continuous.

Check: the restriction of a uniformly continuous function is unif. cont.

$\forall \varepsilon > 0$ want $\exists \delta > 0$ s.t. $d(x_1, x_2) < \delta \Rightarrow d(f(x_1), f(x_2)) < \varepsilon$ $\forall x_1, x_2 \in X \setminus \{x\}$

can find such a δ that works for all $x_1, x_2 \in X$, even better!

Suppose f is unif. cont.

Case 1: $\exists\, \varepsilon > 0$ s.t. $D_\varepsilon(x) = \{x\}$.

Case 2: $\forall\, \varepsilon > 0$ $\exists\, x' \neq x$ $x' \in D_\varepsilon(x)$.

If **Case 1**: choose $y \in Y \neq \emptyset$ arbitrarily.

set $\tilde{f}(x) = y$.

$\delta = \varepsilon$ always works to show continuity at x.

Case 2: By today's philosophy, can find a sequence (x_n) in $X \setminus \{x\}$ s.t. $\lim_{n \to \infty} x_n = x$

Claim: $f(x_n)$ converge in Y. (in this case, we'll define $f(x) = \lim_{n \to \infty} f(x_n)$)

We'll first show that $f(x_n)$ is a Cauchy subsequence

given $\varepsilon > 0$, by unif cont. $\exists\, \delta$ s.t.

if $d(x', x'') < \delta \Rightarrow d(f(x'), f(x'')) < \varepsilon$

given this δ, $\exists\, N$ s.t. $n, m \geq N \Rightarrow d(x_n, x_m) < \delta$

$\Rightarrow d(f(x_n), f(x_m)) < \varepsilon$

Since Y is complete limit makes sense

$$f(x) = \lim_{n\to\infty} f(x_n)$$

Is this well defined?

if $\lim x_n' = x = \lim x_n$ then is

$\lim f(x_n') = \lim f(x_n)$

yes because can show $\forall \varepsilon > 0 \ \exists N > 0$ s.t.

$n \geq N$ then $d(f(x_n), f(x_n')) < \varepsilon$

why? by unif cont. as above

given $\varepsilon > 0$, by unif cont. $\exists \delta$ s.t.

if $d(x', x'') < \delta \Rightarrow d(f(x'), f(x'')) < \varepsilon$

because both $\lim x_n = x = \lim x_n'$, given $\delta > 0$ can find

N'' s.t. $n \geq N''$ then $d(x_n, x) \leq \frac{\delta}{2}$ $d(x_n', x) \leq \frac{\delta}{2}$

$\Rightarrow \delta(x_n, x_n') \leq \delta \Rightarrow \delta(f(x_n), f(x_n')) < \varepsilon$

About the exam:

- Closed book, no calculators, phones
- 1 page $8\frac{1}{2} \times 11$ handwritten "cheat sheet" (both sides)
- Any errors found on exam w/in first 20 minutes = same amount of extra credit!

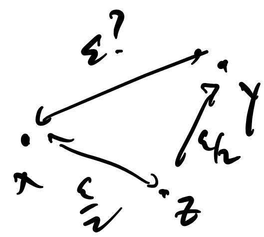
General principle (1)

1. if points in S are arbitrarily close to another pt x

   then some of them converge to x.

2. Contradiction / draw a picture / guess & check / "use your words"

3. $\frac{\varepsilon}{2}$ + Δ inequality



4. ?