$|G| = 150 = 2\cdot 3\cdot 5^2$

show $\exists\, N \triangleleft G$ propr, $5 \mid |N|$.

$n_5 \equiv_5 1$ $n_5 \mid 6$ $n_5 = \underset{\checkmark}{1}$ or 6

$G \circlearrowright \mathrm{Syl}_5 G$

$G \xrightarrow{\varphi} S_6$ $K = \ker\varphi$

$5 \mid |K|$

$\ker\varphi < \underset{25}{\mathrm{Stab}_G P}$ $6 = |\mathrm{Syl}_5 G| = \dfrac{\overset{=150}{|G|}}{\underset{25=}{|\mathrm{Stab}\, P|}}$

Cohomology main points

H^0 = fixed stuff $H^0(G, X) = X^G$ (makes sense for a G-set X)

H^1 = crossed homs$/\sim$ $\varphi : G \to N$ N a G-group

$\varphi(gh) = \underbrace{\varphi(g)}_{N}\, g\cdot \underbrace{\varphi(h)}_{N}$

$\varphi(g)\, {}^{g}\varphi(h)$

H^2 = Noether factor sets$/\sim$ $\varphi : G\times G \to A$ A G-module.

classify extensions $1 \to A \to \square \to G \to 1$

"D is an extension of G by A"

Form a LE sequence

$$1 \to A \to B \to C \to 1$$

$$\begin{tikzcd}
1 \arrow[r] & H^0(G,A) \arrow[r] & H^0(G,B) \arrow[r] & H^0(G,C) \arrow[dll, rounded corners] \\
 & H^1(G,A) \arrow[r] & H^1(G,B) \arrow[r] & H^1(G,C) \arrow[dll, rounded corners] \\
 & H^2(G,A) \arrow[r] & H^2(G,B) \arrow[r] & H^2(G,C) \arrow[dll, rounded corners] \\
 & \cdots & &
\end{tikzcd}$$

stops here if C is only a G-set

stops here if A is not Abelian but all groups

stops here if A is G-mod But B is nonab.

if B (& everything else) Abelian, goes forever.

A a G-module if $(|G|,|A|)=1$

then $H^n(G,A)=0$ for $n \geq 1$.

Should know definitions of nilpotent & solvable.

composition series

chief series $(X=G)$

Hall's Theorem

Schur-Zassenhaus

Jordan-Hölder

ngular.　　　　　　　　# rot. symmetries.

hecaton icosachoron | dodecacontachoron |

# of 3 cell faces 120 each a dodecahedron.

$|\text{orbit}| = \dfrac{|G|}{|\text{Stab}|}$　　　　$G \circlearrowright \text{cells}$

$$\underset{120}{|\text{cells}|} = \frac{|G|}{|\text{Stab of a cell}|}$$

Stab of cell $\circlearrowright$ $\overset{a}{\text{cell}}$ (dodecahedron)

$$\text{Stab of cell} = \overset{\text{rot.}}{\text{symm.}} \text{ of cell}$$

Dodecahedron: 12 $\overset{\text{faces}}{\text{sides}}$, each a pentagon

$H = \overset{\text{rot.}}{\text{symm}}$ of Dodeca.

$H \circlearrowright \text{faces}$　　　　$\underset{\underset{12}{\shortparallel}}{\#\text{faces}} = \dfrac{|H|}{|\text{Stab}|}$ $= \overset{\text{rot.}}{\text{symm.}}$ of pentagon $\shortparallel$ 5

$|H| = 60$

$|G| = 120 \cdot 60 = 7{,}200$

$|G| = 90$ $\exists$ elmt of order 9

$= 9 \cdot 2 \cdot 5$

$n_3 = 1, 10$ $n_5 = 1, 6$ $n_2 = 1, 3, 5, 9, 15, 45$

elmts of order 9   elmts of order 5

$10 \cdot 6$ $6 \cdot 4 = 24$

$= 60$

if not normal,
at least 3 elmts of order 2

84 ⇝ 87 ⇝ last 3 elmts
are a subgp of order 3.
normal

$n_2 = 5$ ⇝ 5 elmt of order 2

89