Outline for Graduate Algebra
UPenn, Fall 2024

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Contents

Part I

First semester: noncommutative algebra

Chapter 1

Groups

1.1 Week 1: Group fundamentals

1.1.1 Definitions and examples

[Isa09, Ch. 1]

1. def of functions, injectivity, surjectivity, left vs right notation
2. injective if right invertible, surjective if left invertible, if bijective then inverses are equal
3. permutation groups
4. binary operations, magmas, monoids, loops, groups

1.1.2 Subgroups and cosets

[Isa09, Ch. 2]

1.1.3 Homomorphisms

[Isa09, Ch. 3]

1.2 Week 2: Actions and permutations

1.2.1 Group actions

[Isa09, Ch. 4]

1.2.2 Permutation groups

[Isa09, Ch. 6]

1.3 Week 3: Sylow theorems and p-groups

[Isa09, Ch. 5]

1.4 Week 4: Semidirect products and extensions

[Isa09, Ch. 7]

1.5 Week 5: Group cohomology and Schur-Zassenhaus

[Wei94, Section 6.6], [Bro94, Section IV.3]

1.6 Week 6: Solvable and nilpotent groups

[Isa09, Ch. 8]

Chapter 2

X-groups

2.1 Week 7: Composition series

[Isa09, Ch. 10]

1. composition series for groups (with operators), modules
2. jordan-holder, length
3. solvable, supersolvable, nilpotent
4. krull-schmidt

2.2 Week 8: Chain conditions and reducibility

[Isa09, Ch. 11]

1. ascending and descending chain conditions for groups (with operators) and modules
2. complete reducibility/semisimplicity
3. zorn's lemma

Chapter 3

Noncommutative Rings

3.1 Week 9: Basic notions

[Isa09, Ch. 12]

3.2 Week 10: Wedderburn-Artin theory

[Isa09, Ch. 13 and a bit of Ch. 14]

3.3 Week 11: Categories of modules

3.3.1 Tensor products, injectives, projectives, flats

3.4 Week 12: Morita theory

[AF92, Ch. 6]

3.5 Week 13: Homological algebra

3.5.1 Ext and extensions, Tor

3.5.2 Limits

Part II

Second semester: commutative algebra

Chapter 4

Field theory

4.1 Week 1: Ring preliminaries

[Isa09, Ch. 16]

4.1.1 first lecture

1. factorization and UFDs
2. modules over PIDs

4.1.2 second lecture

1. localization in general
2. prime ideals in localizations
3. completions, hensel's lemma
4. primes as transitional to field theory, nakayama reminder

4.2 Week 2: Field extensions and Galois theory

4.2.1 Field extensions

[Isa09, Ch. 17]

4.2.2 Galois theory

[Isa09, Ch. 18] [Jac85, Ch. 4]

1. primitive element [Jac85, Thm 4.28]

2. dedekind's lemma [Jac85, § 4.14]
3. normal basis [Jac85, § 4.14]

4.3 Week 3: Further topics in Galois theory

[Jac89, Ch. 8] and other notes.

4.3.1 Descent and Hilbert 90

1. descent via morita theory
2. crossed homomorphisms, H^1 and twisted forms
3. hilbert 90

4.3.2 Cyclic extensions

1. Kummer theory
2. Artin-Schrier theory

4.4 Week 4: Separable and inseparable extensions

[Isa09, Ch. 19]

4.5 Week 5: Applications

4.5.1 Cyclotomic extensions and geometric constructions

[Isa09, Ch. 20]

4.5.2 Finite fields

[Isa09, Ch. 21]

4.6 Week 6: Norms, traces, discriminants, resultants

[Isa09, Ch. 23]

4.7 Week 7: Transcendental extensions

[Isa09, Ch. 24]

Chapter 5

Commutative (mostly Noetherian) rings

5.1 Week 8: Primary decomposition, height and dimension

[Isa09, Ch. 27], [Row06, Ch. 6]

1. Hilbert basis
2. lasker-noether
3. krull intersection
4. krull's principal ideal theorem
5. krull dimension

5.2 Week 9: Some structural results on rings and modules

[Isa09, Ch. 24], [Row06, Ch. 6]

1. integral extensions
2. noether normalization
3. mention Cohen structure theorem
4. flat = projective = locally free for finitely presented modules

5.3 Week 10: Dedekind domains

[Isa09, Ch. 29]

Chapter 6

Linear algebra

6.1 Week 11: Multilinear algebra

6.1.1 Duals, exterior and symmetric forms

6.1.2 Bilinear and quadratic forms

6.2 Week 12: Canonical forms

Chapter 7

Algebraic geometry

7.1 Week 13: Rings as coordinate rings

Appendix A

other resources

Bibliography

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