<u>Def</u> If $S \subset G$ subset of a group G, we define $\langle S \rangle$ = subgroup generated by S to be the smallest subgroup of G containing S.

$$\langle S \rangle = \bigcap_{S \subset H < G} H$$

<u>Def</u> $S \subset G$ subset, define $W(S)$ "words in S"

to be $\left\{ s_1^{\varepsilon_1} s_2^{\varepsilon_2} \cdots s_r^{\varepsilon_r} \mid s_i \in S,\ \varepsilon_i \in \{\pm 1\} \right\}$

including empty sequence $\emptyset$

Note, there is a natural map

$$W(S) \longrightarrow G \quad \text{"evaluation"}$$

$$s_1^{\varepsilon_1} \cdots s_r^{\varepsilon_r} \longrightarrow \text{product in group}$$

"$s_1^{\varepsilon_1} \cdots s_r^{\varepsilon_r}$"

<u>Claim</u> $\langle S \rangle$ = image of $W(S)$ in G. ✓

moreover, if S any set, G any group

$S \to G$ any map, the above gives an ext from $W(S) \to G$

Def Let S be a set, $W(S)$ is a set.

we say $w \sim w'$ $\quad (w, w' \in W(S))$

if whenever we have a group G

and a map $S \to G$, w, w' map to same elmt of G.

Def $F(S) = W(S)/\sim$.

Claim: $F(S)$ is a group under the operation

$w \cdot w' = ww'$ (concatenation)

$w_1 \sim w_2$ $\qquad w_1 w_1' \sim w_2 w_2'$? ✓

$w_1' \sim w_2'$

Def $R(S)$ "reduced words"

"$\{ s_1^{\varepsilon_1} \cdots s_n^{\varepsilon_n} \mid \text{if } s_i = s_{i+1} \text{ then } \varepsilon_i = \varepsilon_{i+1} \}$"

Claim

$$\begin{tikzcd} R(S) \arrow[r] \arrow[rr, bend right, "\text{is bijective.}"'] & W(S) \arrow[r] & W(S)/\sim \end{tikzcd}$$

Surjectivity: suppose $w \in W(S)$ is minimal length such that $w \not\sim w'$ any $w' \in R(S)$.

in w have

$$w = w' s_i^{\varepsilon} s_{i+1}^{-\varepsilon} w'' \sim w' w''$$ contradicts minimality

injective:

if $w, w' \in R(S)$ $w \neq w'$

$G = \mathrm{Sym}(R(S))$

for $s \in S$, consider permutation

$$\varphi_s(s_1^{\varepsilon_1} \cdots s_n^{\varepsilon_n}) = \begin{cases} s' s_1^{\varepsilon_1} \cdots s_n^{\varepsilon_n} & \text{if } s \neq s_1 \text{ or } \varepsilon_1 = 1 \\ s_2^{\varepsilon_2} \cdots s_n^{\varepsilon_n} & \text{else.} \end{cases}$$

$w'' \in W(S)$ define

$$\varphi_{w''} = \varphi_{s_1}^{\varepsilon_1} \cdots \varphi_{s_n}^{\varepsilon_n}$$

now actions of w & w' on the empty reduced word gives w & w' back which are different $\square$.