(1) Let $x(t) = 7t$ and $y(t) = 5\sin(3\pi t)$ be analog signals on the interval $0 \le t \le 1$ and set $z(t) = x(t) + 2y(t)$.

(a) Sample $x(t)$ and $y(t)$ at times $t = 0, 0.25, 0.5, 0.75$ to produce a sample vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^4$. Verify that $\mathbf{x} + 2\mathbf{y}$ is the vector $\mathbf{z}$ obtained by sampling $z(t)$ at these same times. *Sampling is linear.*

(b) For $\mathbf{u} \in \mathbb{R}^4$ let $q(\mathbf{u})$ denote the vector whose components are obtained by applying the `floor` function to each component of $\mathbf{u}$. If $\mathbf{z}$, $\mathbf{x}$, and $\mathbf{y}$ are the vectors in (a), show that $q(\mathbf{z}) \neq q(\mathbf{x}) + 2q(\mathbf{y})$. *Quantization is nonlinear.*

(2) Let $\mathbb{F}$ be $\mathbb{R}$ or $\mathbb{C}$. Show that the following sets V with the given operations of vector addition and scalar multiplication satisfy the vector space axioms A1-A5 in Section 1.3.

(a) $V = \mathbb{F}^{m \times n}$, with the usual addition and scalar multiplication of matrices.

(b) Let X be the interval of real numbers $a \le x \le b$ and $V = C_{\mathbb{F}}(X)$ (all continuous $\mathbb{F}$-valued functions on X) with the usual addition and scalar multiplication of functions. (HINT: From calculus $cf(x)$ and $f(x) + g(x)$ are continuous if $f(x)$ and $g(x)$ are continuous and c is a constant.)

(c) Let V be the set of all polynomials $f(x) = a_0 + a_1 x + \cdots + a_n x^n$ with $a_j \in \mathbb{F}$ and n any nonnegative integer, with the usual addition and scalar multiplication of functions.

(3) Determine whether the following sets M of matrices are subspaces of $\mathbb{R}^{2\times 2}$. When this is the case, find a basis for M.

(a) M consists of all 2×2 diagonal matrices.

(b) M consists of all 2×2 upper triangular matrices (zero below diagonal).

(c) M consists of all symmetric 2×2 matrices.

(d) M consists of all 2×2 matrices A with $\det A = 0$.

(4) Let V be the real vector space of the continuous real-valued functions on the interval $-1 \le x \le 1$. Let U be the set of all functions $f \in V$ such that $f(-1) = 2f(1)$. Prove that U is a subspace of V.