Applied Algebra, Homework 4

1. Suppose $f(t) = 2^x$ on the interval $[0, 4)$ and extended periodically (with period 4). Consider the sampled function $f[j]$ where $j \in \mathbb{Z}/4\mathbb{Z}$. Find $\hat{f}[1], \hat{f}[2]$ (show your work!).
2. Consider the space $\ell_{\mathbb{C}}(\mathbb{Z}/4\mathbb{Z}) = \mathbb{C}^4$ of sampled functions, with basis e_j defined by $e_j[k] = \delta_{j,k}$. Define a linear transformation $S : \mathbb{C}^4 \to \mathbb{C}^4$ by $Se_0 = e_1, Se_1 = e_2, Se_2 = e_3, Se_3 = e_0$. Show that the basic discrete wavefunctions E_j, $j = 0, 1, 2, 3$ are eigenvectors for S, and compute their eigenvalues.