In general - open meeting at 12 - will formally start at 12:20
will go for ≤ 1 hour from there.

Videos & worksheets most tues / fridays
worksheet - only graded for completion.

Weekly assignments via Sakai Friday → Thursday

Exams - open book takehome via Sakai.

Wavelet transform

signal → trend = simplified / smoothed / de-noised
signal → detail = short scale fluctuation / noise

$$t[k] = \frac{x[2k] + x[2k+1]}{2}$$

← represent value "in the vicinity at $x[2k+1]$"

$$d[k] = \frac{x[2k] - x[2k+1]}{2}$$

← represents error in above.

$$x \longmapsto \begin{bmatrix} t \\ d \end{bmatrix} \qquad N = 2m$$

$T_a : \mathbb{R}^N \longleftrightarrow \mathbb{R}^N$

T_a = analysis matrix

$T_s = T_a^{-1}$ = sythesis matrix.

Approximaton scheme $\quad x \rightsquigarrow$ series of values at odd pts t

$\rightsquigarrow$ errors d

$\mathbb{R}^{2m} \to \mathbb{R}^m$

$x \mapsto t$

$x \mapsto d$

$$\underset{\text{standard}}{\mathbb{R}^{2m}} \underset{T_s}{\overset{T_a}{\rightleftarrows}} \underset{\text{wavelet}}{\mathbb{R}^{2m}}$$

$$x \longrightarrow \begin{bmatrix} \square \\ \square \\ \\ \\ \end{bmatrix}$$

how much wavelet #1

" #2

wavelet #1 in standard basis $\longleftarrow\!\!\shortmid \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ ← wavelet #1

<u>Haar wavelet</u>: guess for near $x[2k+1] \sim \dfrac{x[2k] + x[2k+1]}{2}$

error $= (\text{above}) - x[2k+1]$

$N=4$ $\qquad \underbrace{e_0 \quad e_1} \quad \underbrace{e_2 \quad e_3}$

$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix} = e_0 \longmapsto t = \begin{bmatrix}1/2\\0\end{bmatrix} \qquad d = \begin{bmatrix}1/2-0\\0-0\end{bmatrix} = \begin{bmatrix}1/2\\0\end{bmatrix}$$

$$\begin{bmatrix}0\\1\\0\\0\end{bmatrix} = e_1 \longmapsto t = \begin{bmatrix}1/2\\0\end{bmatrix} \qquad d = \begin{bmatrix}1/2-1\\0\end{bmatrix} = \begin{bmatrix}-1/2\\0\end{bmatrix}$$

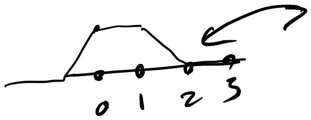
$$e_2 \longmapsto t = \begin{bmatrix}0\\1/2\end{bmatrix} \qquad d = \begin{bmatrix}0\\1/2\end{bmatrix}$$

$$e_3 \longmapsto t = \begin{bmatrix}0\\1/2\end{bmatrix} \qquad d = \begin{bmatrix}0\\-1/2\end{bmatrix}$$

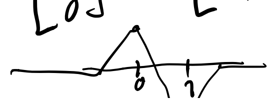
$$T_a = \left[\begin{array}{c|c|c} \text{trend}(e_0) & \text{trend}(e_1) & \\ & & \cdots \\ \text{detail}(e_0) & \text{det}(e_1) & \end{array}\right]$$

$$= \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & -1/2 \end{bmatrix}$$

$$T_s = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix} = T_a^{-1}$$



$$\begin{bmatrix}1\\1\\0\\0\end{bmatrix} \qquad \begin{bmatrix}0\\0\\1\\1\end{bmatrix} \qquad \begin{bmatrix}1\\-1\\0\\0\end{bmatrix} \qquad \begin{bmatrix}0\\0\\1\\-1\end{bmatrix}$$



Process #1 for wavelet design

- Think of a good "prediction method" to describe trend

rest "follows" from this decision.

Problems:

- Want process to be computationally simple
- Want "energy preservation"

$$\text{"} \int |f|^2 ds \text{"} = \text{"} \int |t|^2 ds + \int |d|^2 ds$$

- want it to "work well"

trend – described by a filter removing high freq info

detail – removes low freq info