(Worksheet from lecture 13)

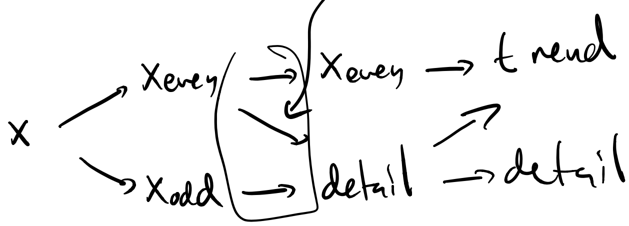
$$t[k] = \frac{1}{4}x[2k] + \frac{3}{4}x[2k+1] = \boxed{\frac{1}{4}x_{even}[k] + \frac{3}{4}x_{odd}[k]}$$

$$d[k] = t[k] - x[2k+1] = \frac{1}{4}x_{even}[k] + \frac{3}{4}x_{odd}[k] - x_{odd}[k]$$

$$t[k] = d[k] + x_{odd}[k] \qquad T_s = T_a^{-1}$$

$$= \frac{1}{4}x_{even} - \frac{1}{4}x_{odd} = -\frac{1}{4}(x_{odd} - x_{even})$$

$$T_a x = \begin{bmatrix} t \\ d \end{bmatrix} \qquad \begin{bmatrix} I & 0 \\ -I & I \end{bmatrix}$$



$$\text{trend} = x_{even} + \square \cdot (x_{odd} - x_{even})$$

$$= x_{even} + \frac{3}{4}(x_{odd} - x_{even})$$

$$= \frac{1}{4}x_{even} + \frac{3}{4}x_{odd}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{4} \end{bmatrix} \begin{bmatrix} I & \frac{3}{4}I \\ 0 & I \end{bmatrix} \begin{bmatrix} I & 0 \\ -I & I \end{bmatrix} \boxed{\text{split}} = T_a$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1/4 \end{bmatrix} \begin{bmatrix} 1 & 3/4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = T_a$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{bmatrix} 1 & -3/4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -4 \end{bmatrix} = T_s = T_a^{-1}$$

Exam Information

There will be a choice

"project"
relatively
open ended
w/ concrete things
to try to do.

exam(s)

$N = 2m$

$$\begin{array}{c} x \\ \text{signal} \end{array} \begin{array}{l} \nearrow x_{even} \\ \searrow x_{odd} \end{array}$$

$$\begin{bmatrix} x_0 \\ \vdots \\ x_{2m-1} \end{bmatrix} \longmapsto \begin{bmatrix} x_0 \\ x_2 \\ \vdots \\ x_{2m-2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \\ \vdots \\ x_{2m-1} \end{bmatrix} \longmapsto \begin{bmatrix} x_0 \\ x_2 \\ \vdots \\ x_{2m-2} \\ x_1 \\ x_3 \\ \vdots \\ x_{2m-1} \end{bmatrix}$$

$\boxed{\text{split}}$

$$\mathbb{R}^{2m} \longrightarrow \mathbb{R}^{2m}$$

$$e_{2k} \longmapsto e_k \qquad k < m$$

$$e_{2k+1} \longmapsto e_{m+k}$$

$$\xleftarrow{\boxed{\text{merge}}}$$

$$\begin{bmatrix} I & 0 \\ -I & I \end{bmatrix} \begin{bmatrix} x_{even} \\ x_{odd} \end{bmatrix} = \begin{bmatrix} x_{even} \\ x_{odd} - x_{even} \end{bmatrix}$$

$$\begin{bmatrix} I & 0 \\ -I & I \end{bmatrix} \underbrace{\boxed{\text{split}}\ e_0}_{e_0} =$$

$$\begin{bmatrix} I & 0 \\ -I & I \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ \hline 0 \\ \vdots \\ 0 \end{bmatrix} \begin{matrix} \leftarrow x_{en} \\ \\ \leftarrow x_{odd} \end{matrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ \hline 0 \\ \vdots \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = e_0 - e_m$$

$$\begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix} e_k = e_{k+m}$$

$k < m$

$$\begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{bmatrix} \leftarrow 2k$$

$k < m$

$$e_{2k} \xrightarrow{\text{split}} e_k \leftarrow \text{sitting in first } 1/2 \text{ of vector}$$

$$e_{2k} = x \qquad \begin{bmatrix} x_{even} \\ x_{odd} \end{bmatrix}$$

$2k < 2m = n$

$$x_{even} = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{bmatrix} \leftarrow k\text{th place}$$

$$x_{odd} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} I & 0 \\ -I & I \end{bmatrix} e_{2k} = \begin{bmatrix} I & 0 \\ -I & I \end{bmatrix} \begin{bmatrix} x_{even} \\ x_{odd} \end{bmatrix}$$

$$= \begin{bmatrix} x_{even} \\ x_{odd} - x_{even} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \hline 0 \\ \vdots \\ -1 \\ \vdots \\ 0 \end{bmatrix} \begin{matrix} \leftarrow k\text{th} \\ \\ \\ \leftarrow k\text{th place in } 2^{nd} \text{ half} \end{matrix}$$

v

$\parallel$

$e_k - e_{k+m}$

2. expression for DUP part of CDF(2,2)

CDF(2,2)

$$D = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 1/\sqrt{2} \end{bmatrix} \qquad U = \begin{bmatrix} I & \frac{1}{4}I + \frac{1}{4}S \\ 0 & I \end{bmatrix}$$

$$P = \begin{bmatrix} I & 0 \\ -\frac{1}{2}I - \frac{1}{2}S^{-1} & I \end{bmatrix}$$

$$UP = \begin{bmatrix} I & \frac{1}{4}I + \frac{1}{4}S \\ 0 & I \end{bmatrix} \begin{bmatrix} I & 0 \\ -\frac{1}{2}I - \frac{1}{2}S^{-1} & I \end{bmatrix}$$

$$= \begin{bmatrix} I + \frac{1}{8}\left(-I - S^{-1} - S - I\right) & \frac{1}{4}(I + S) \\ -\frac{1}{2}\left(I + S^{-1}\right) & I \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{4}I - \frac{1}{8}(S^{-1}+S) & \frac{1}{4}(I+S) \\ -\frac{1}{2}(I+S^{-1}) & I \end{bmatrix}$$

$$\begin{bmatrix} X_{even} \\ X_{odd} \end{bmatrix}$$

$$T_s = T_a^{-1} = \left(D\,U\,P\,\boxed{\text{split}} \right)^{-1}$$

$$= \boxed{\text{split}}^{-1} P^{-1} U^{-1} D^{-1}$$

$$T_s = \boxed{\text{merge}}\; P^{-1}\, U^{-1}\, D^{-1}$$

$$D^{-1} = \begin{bmatrix} 1/\sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} I & 0 \\ \frac{1}{2}(I+S^{-1}) & I \end{bmatrix} \qquad U^{-1} = \begin{bmatrix} I & -\frac{1}{4}(I+S) \\ 0 & I \end{bmatrix}$$

$$e_{m+j} = e_k \qquad m \le k < 2m$$

how to find wavelet basis?

$$T_s e_k \qquad \begin{aligned} &0 \le k < m \leftarrow \text{trend} \\ &m \le k < 2m \leftarrow \text{detail} \end{aligned}$$

Suppose $0 \le k < m$ consider $T_s e_k$

$$T_s e_k = \boxed{\text{merge}}\, P^{-1} U^{-1} D^{-1} e_k$$

$$= \boxed{\text{merge}}\, P^{-1} U^{-1} \left(\frac{1}{\sqrt{2}}\right) e_k$$

$$= \left(\frac{1}{\sqrt{2}}\right) \boxed{\text{merge}}\, P^{-1} \underbrace{U^{-1} e_k}_{e_k \text{ again}}$$

$$\begin{bmatrix} I & U \\ 0 & I \end{bmatrix} e_k \qquad \begin{bmatrix} * \\ \hline 0 \end{bmatrix} \qquad = e_k$$

$$= \left(\frac{1}{\sqrt{2}}\right) \boxed{\text{merge}}\, \underbrace{P^{-1} e_k}$$

$$\begin{bmatrix} I & 0 \\ \frac{1}{2}(I + S^{-1}) & I \end{bmatrix} \underbrace{\begin{bmatrix} * \\ \hline 0 \end{bmatrix}}_{e_k} = \begin{bmatrix} * \\ \frac{1}{2}\boxed{*} + \frac{1}{2} S^{-1} \underbrace{*} \end{bmatrix}$$

e_k

e_{k+m}

e_{k+m-1}

($k=0$

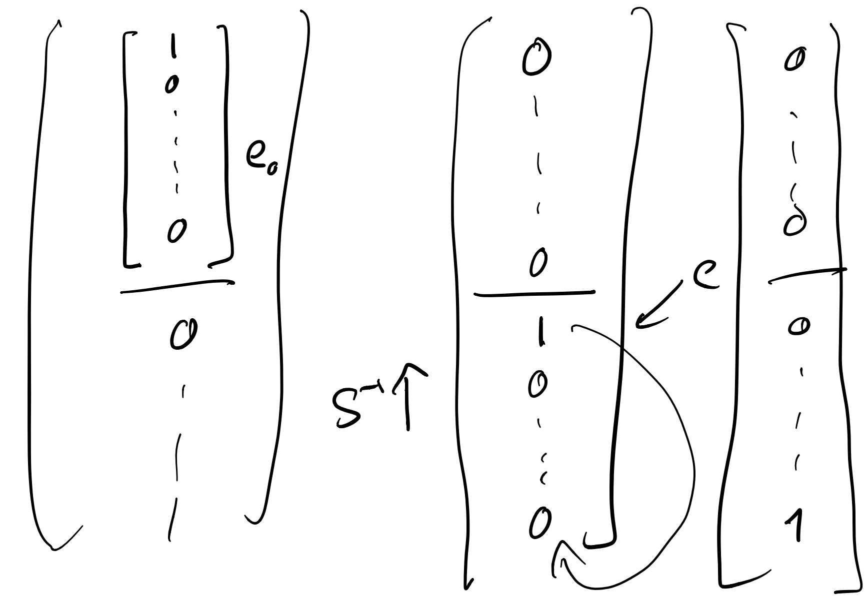
e_{2m-1})

concretely:

$P^{-1}U^{-1}D^{-1}e_k$ $\qquad$ $0 \le k < m$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(e_k + \frac{1}{2} e_{k+m} + \frac{1}{2} e_{k+m-1}\right)$$

$\boxed{\text{merge}}$



$$\left(\frac{1}{\sqrt{2}}\right)\left(e_{2k} + \frac{1}{2} e_{2k+1} + \frac{1}{2} e_{2k-1}\right)$$

kth trend wavelet basis vector.