#3 from worksheet

$$T_a = D\,U\,P\,\boxed{\text{split}}$$

$$D = \begin{bmatrix} \sqrt{2}I & 0 \\ 0 & 1/\sqrt{2}\,I \end{bmatrix}$$

2m × 2m (block) matrix

$$U = \begin{bmatrix} I & \frac{1}{4}I + \frac{1}{4}S \\ 0 & I \end{bmatrix}$$

$$P = \begin{bmatrix} I & 0 \\ -\frac{1}{2}I - \frac{1}{2}S^{-1} & I \end{bmatrix}$$

$$T_s = \boxed{\text{split}}^{-1} P^{-1} U^{-1} D^{-1}$$

$$= \boxed{\text{merge}} \begin{bmatrix} I & 0 \\ \frac{1}{2}I + \frac{1}{2}S^{-1} & I \end{bmatrix} \begin{bmatrix} I & -\frac{1}{4}I - \frac{1}{4}S \\ 0 & I \end{bmatrix} \begin{bmatrix} 1/\sqrt{2}\,I & 0 \\ 0 & \sqrt{2}I \end{bmatrix}$$

$T_s e_j = j^{th}$ column of T_s = wavelet bases

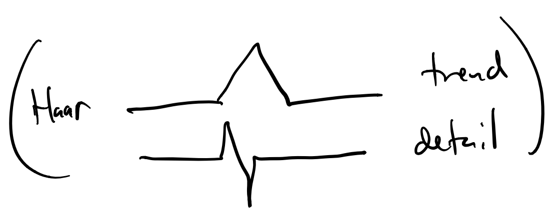
$j = 0, 1, 2, \ldots, 2m-1$

$$x \underset{T_s}{\overset{T_a}{\rightleftarrows}} \begin{bmatrix} s \\ d \end{bmatrix}$$

trend

detail

$$\begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = e_j$$



$T_s \begin{bmatrix} \text{"}e_k\text{"} \\ -- \\ 0 \end{bmatrix}$ to mean 1 in kth entry of first part of my vector

$T_s \begin{bmatrix} 0 \\ -- \\ \text{"}e_k\text{"} \end{bmatrix}$ 1 in k th entry of second part of vector.

$$\begin{bmatrix} \text{"}e_k\text{"} \\ -- \\ 0 \end{bmatrix} = e_k \qquad \begin{bmatrix} 0 \\ -- \\ \text{"}e_k\text{"} \end{bmatrix} = e_{k+m}$$

meaning: "e_k" kth basis vector in $\mathbb{C}^m$

e_k - - - - - - - $\mathbb{C}^{2m}$

$$T_s = \boxed{\text{split}}^{-1} P^{-1} U^{-1} D^{-1}$$

$$= \boxed{\text{merge}} \underbrace{\begin{bmatrix} I & 0 \\ \frac{1}{2}I + \frac{1}{2}S^{-1} & I \end{bmatrix}}_{P^{-1}} \underbrace{\begin{bmatrix} I & -\frac{1}{4}I - \frac{1}{4}S \\ 0 & I \end{bmatrix}}_{U^{-1}} \underbrace{\begin{bmatrix} 1/\sqrt{2}\, I & 0 \\ 0 & \sqrt{2}\, I \end{bmatrix}}_{D^{-1}}$$

Let's look at the kth detail wavelet in basis.

$$T_s e_{m+k} = T_s \begin{bmatrix} 0 \\ -- \\ \text{"}e_k\text{"} \end{bmatrix}$$

Process: first work out $D^{-1} \begin{bmatrix} 0 \\ -- \\ \text{"}e_k\text{"} \end{bmatrix}$ ✓

then apply u^{-1}, then apply P^{-1}, then apply $\boxed{\text{merge}}$

$$D^{-1} = \begin{bmatrix} \cancel{1/}\sqrt{2}\,I & 0 \\ 0 & \sqrt{2}\,I \end{bmatrix} \begin{bmatrix} 0 \\ --- \\ \text{"}e_k\text{"} \end{bmatrix} = \begin{bmatrix} 0 \\ --- \\ \sqrt{2}\,\text{"}e_k\text{"} \end{bmatrix} = \sqrt{2} \begin{bmatrix} 0 \\ -- \\ \text{"}e_k\text{"} \end{bmatrix}$$

$$\underset{u^{-1}}{\begin{bmatrix} I & -\frac{1}{4}I - \frac{1}{4}S \\ 0 & I \end{bmatrix}} \cdot \left(\sqrt{2} \begin{bmatrix} 0 \\ --- \\ \text{"}e_k\text{"} \end{bmatrix} \right)$$

$$= \sqrt{2} \begin{bmatrix} \bullet \\ \bullet \end{bmatrix} = \sqrt{2} \begin{bmatrix} -\frac{1}{4} I \,\text{"}e_k\text{"} - \frac{1}{4} S \,\text{"}e_k\text{"} \\ \text{"}e_k\text{"} \end{bmatrix}$$

$$= \sqrt{2} \begin{bmatrix} -\frac{1}{4}\text{"}e_k\text{"} - \frac{1}{4}\text{"}e_{k+1}\text{"} \\ \text{"}e_k\text{"} \end{bmatrix}$$

$$\text{"}e_m\text{"} = \text{"}e_0\text{"}$$

$$\sqrt{2} \underset{P^{-1}}{\begin{bmatrix} I & 0 \\ \frac{1}{2}I + \frac{1}{2}S^{-1} & I \end{bmatrix}} \begin{bmatrix} -\frac{1}{4}\text{"}e_k\text{"} - \frac{1}{4}\text{"}e_{k+1}\text{"} \\ \text{"}e_k\text{"} \end{bmatrix}$$

$$= \sqrt{2} \begin{bmatrix} \bullet \\ \bullet \end{bmatrix} = \sqrt{2} \begin{bmatrix} -\frac{1}{4}\text{"}e_k\text{"} - \frac{1}{4}\text{"}e_{k+1}\text{"} \\ -\frac{1}{8}\left(\text{"}e_k\text{"} + \text{"}e_{k+1}\text{"} + \text{"}e_{k-1}\text{"} + \text{"}e_k\text{"} \right) + \text{"}e_k\text{"} \end{bmatrix}$$

$$= \frac{\sqrt{2}}{4} \begin{bmatrix} -\text{"}e_k\text{"} - \text{"}e_{k+1}\text{"} \\ -\frac{1}{2}\text{"}e_{k-1}\text{"} - \frac{1}{2}\text{"}e_{k+1}\text{"} + 3\text{"}e_k\text{"} \end{bmatrix}$$

$\boxed{\text{merge}}$ practice:

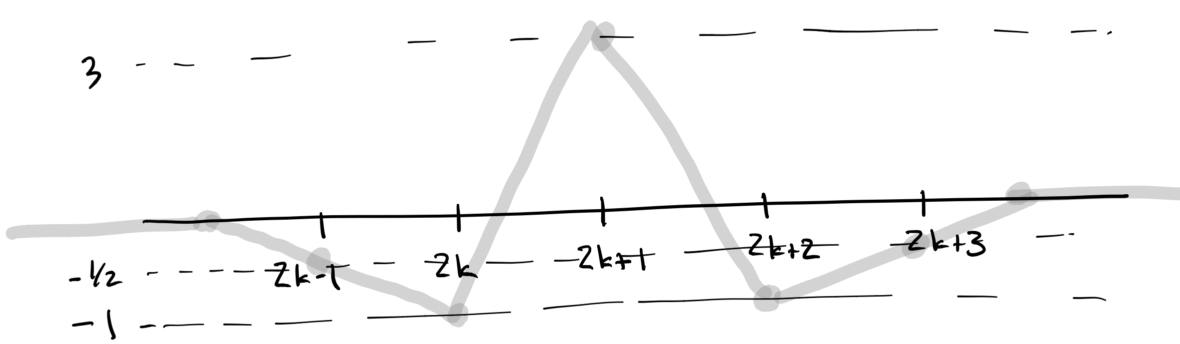
$$\boxed{\text{merge}} \begin{bmatrix} \text{"}e_k\text{"} \\ \hline 0 \end{bmatrix} = e_{2k}$$

$$\boxed{\text{merge}} \begin{bmatrix} 0 \\ \hline \text{"}e_k\text{"} \end{bmatrix} = e_{2k+1}$$

$$\boxed{\text{merge}} \frac{\sqrt{2}}{4} \begin{bmatrix} -\text{"}e_k\text{"} - \text{"}e_{k+1}\text{"} \\ -\frac{1}{2}\text{"}e_{k-1}\text{"} - \frac{1}{2}\text{"}e_{k+1}\text{"} + 3\text{"}e_k\text{"} \end{bmatrix}$$

$$\frac{\sqrt{2}}{4}\left(-e_{2k} - e_{2k+2} - \frac{1}{2} e_{2k-1} - \frac{1}{2} e_{2k+3} + 3\, e_{2k+1} \right)$$

kth detail wavelet!



e_{2k+3} $\qquad N=8$

$k=0 \quad e_3 \ \checkmark$

$k=3 \quad e_{6+3} = e_9 = e_1$

$N=8$

$T_s\begin{bmatrix} \text{"}e_0\text{"} \\ \hline 0 \end{bmatrix} = T_s e_0 \qquad T_s e_1 \qquad T_s e_2 \qquad \left.T_s e_3\right\} \; T_s\begin{bmatrix} * \\ \hline 0 \end{bmatrix}$

$T_s e_4 = T_s\begin{bmatrix} 0 \\ \hline \text{"}e_0\text{"} \end{bmatrix} \qquad T_s e_5 \qquad T_s e_6 = \begin{bmatrix} 0 \\ \hline \text{"}e_2\text{"} \end{bmatrix} \qquad \left.T_s e_7\right\} \; T_s\begin{bmatrix} 0 \\ \hline * \end{bmatrix}$

$k = 0, 1, \ldots, 7$

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2. DUP

$$UP = \begin{bmatrix} I & \frac{1}{4}I + \frac{1}{4}S \\ 0 & I \end{bmatrix} \begin{bmatrix} I & 0 \\ -\frac{1}{2}I - \frac{1}{2}S^{-1} & I \end{bmatrix}$$

$$= \begin{bmatrix} \; & \; \\ \; & \; \end{bmatrix} = \begin{bmatrix} I + \left(\frac{1}{4}I + \frac{1}{4}S\right)\left(-\frac{1}{2}I - \frac{1}{2}S^{-1}\right) & \frac{1}{4}I + \frac{1}{4}S \\ -\frac{1}{2}I - \frac{1}{2}S^{-1} & I \end{bmatrix}$$

$$
= \begin{bmatrix} I - \frac{1}{8}\left(I + S^{-1} + S + \overbrace{S \cdot S^{-1}}^{I}\right) & \frac{1}{4}I + \frac{1}{4}S \\ -\frac{1}{2}I - \frac{1}{2}S^{-1} & I \end{bmatrix}
$$

$$
= \begin{bmatrix} I - \frac{1}{4}I - \frac{1}{8}S^{-1} - \frac{1}{8}S & \frac{1}{4}I + \frac{1}{4}S \\ -\frac{1}{2}I - \frac{1}{2}S^{-1} & I \end{bmatrix}
$$

$$
= \begin{bmatrix} \frac{3}{4}I - \frac{1}{8}S^{-1} - \frac{1}{8}S & \frac{1}{4}I + \frac{1}{4}S \\ -\frac{1}{2}I - \frac{1}{2}S^{-1} & I \end{bmatrix} = UP
$$