Applied Algebra Worksheet for lecture 16

Comments, questions and feedback due before class on Monday, April 6 (before 7am).

Completed worksheets due on Thursday, April 9.

Consider the potential wavelet transformation given by the following steps:

Start by breaking up your signal into even and odd parts

$[x_{\text{even}}, x_{\text{odd}}]$

first

Then using $p[k] = -x[2k] + 2x[2k+2]$ as a prediction for the value at $x[2k+1]$, set the detail d to be

$\boxed{d[k] = x_{\text{odd}}[k] - p[k]}$ x_{even} info

$T_a = PU$

or UP?

And let P be the linear transformation taking $[x_{\text{even}}, x_{\text{odd}}]$ to $[x_{\text{even}}, d]$.

lose d

Finally, define the trend to be $s[k] = x_{\text{even}}[k] + d[k] - 2d[k-1]$.

Let U be the linear transformation taking $[x_{\text{even}}, d]$ to $[s, d]$.

second first

Step 1: compute d using x_{even} & x_{odd}

Step 2: compute s using x_{even} & d

$T_a x = \underline{PUx}$

UPx

Problems

1. Write a matrix in block form for the linear transformation P
2. Write a matrix in block form for the linear transformation U
3. Write a matrix in block form for the linear transformation T_a which takes the vector $[x_{\text{even}}, x_{\text{odd}}]$ to $[s, d]$

$T_a = UP$

4. Give an explicit presentation for this matrix in the case $N = 4$

Recall:

for CDF(2,2)   Haar

started w/ prediction

$$\left.\begin{array}{l} \text{CDF}(2,2)\quad x[2k+1] \sim \frac{1}{2}x[2k] + \frac{1}{2}x[2k+2] \\ \text{Haar}\quad x[2k+1] \sim x[2k] \end{array}\right\} P$$

$$\left.\begin{array}{l}\text{Daub4 started w/ "trend"} \\ \text{w/ update matrix}\end{array}\right\} U$$

2. Find U = update matrix.

$$U\begin{bmatrix} X_{even} \\ d \end{bmatrix} = \begin{bmatrix} s \\ d \end{bmatrix}$$

$$U = \begin{bmatrix} u_{00} & u_{01} \\ u_{10} & u_{11} \end{bmatrix}$$

$$\boxed{\begin{bmatrix} u_{00} & u_{01} \end{bmatrix}\begin{bmatrix} X_{even} \\ d \end{bmatrix} = s}$$

$$\boxed{\begin{bmatrix} u_{10} & u_{11} \end{bmatrix}\begin{bmatrix} X_{even} \\ d \end{bmatrix} = d}$$

$$[u_{10} \quad u_{11}] = [0 \quad I]$$

$$s = u_{00} x_{even} + u_{01} d$$

u_{ij} = linear transformations on m-dim'l v.space. $\mathbb{C}^m$

$$\boxed{s[k] = x_{even}[k] + d[k] - 2d[k-1]}$$

$$u_{00} = I$$

$$u_{00} x_{even} = x_{even}$$

$$u_{01} d = d - 2Sd = (I - 2S)d$$

$$u_{01} = I - 2S$$

$$d = \text{vector} = \begin{bmatrix} d[0] \\ d[1] \\ \vdots \\ d[m-1] \end{bmatrix} = d[0]\text{"}e_0\text{"} + d[1]\text{"}e_1\text{"} + \dots + d[m-1]\text{"}e_{m-1}\text{"}$$

$$S\text{"}e_k\text{"} = \text{"}e_{k+1}\text{"}$$

$$Sd = d[0]\text{"}e_1\text{"} + d[1]\text{"}e_2\text{"} + \dots + d[m-2]\text{"}e_{m-1}\text{"} + d[m-1]\text{"}e_0\text{"}$$

$$Sd = \begin{bmatrix} d[m-1] \\ d[0] \\ d[1] \\ \vdots \\ d[m-2] \end{bmatrix}$$

$(Sd)[k]$ call it "e_k"

" $d[k-1]$

$$(Sd)[k] = d[k-1]$$

$$U = \begin{bmatrix} I & (I-2S) \\ 0 & I \end{bmatrix}$$