?

$$T_a = \left[\begin{array}{cccc} 3 & \square & 2 & \boxed{-2} \\ \square & -2 & \boxed{3} & -1 \\ \hdashline 1 & 5 & 2 & \square \\ \boxed{2} & -1 & \square & \square \end{array}\right]$$

"scaling vector"

$$T_a = \left[\begin{array}{cccc} u_0 & u_1 & \cdots & u_m \\ & & \cancel{\quad} & \\ \hdashline v_0 & v_1 & \cdots & v_m \\ & & \cancel{\quad} & \end{array}\right]$$

determined

determined

"wavelet vector"

scaling → $\frac{1}{2}$ $\frac{1}{2}$ 0 ——— 0

wavelet → $\frac{1}{2}$ $-\frac{1}{2}$ 0 ——— 0

$$\begin{bmatrix} I+S & S^{-1}+S \\ \cdot & \cdot \end{bmatrix}$$

#1 | Step 1 : write down T_a

Step 2 : grab 1st (0th) row → scaling vector

Step 3 : " -- mth row ⇝ wavelet vector.

$$x \xrightarrow{\boxed{\text{split}}} \begin{bmatrix} x_{even} \\ x_{odd} \end{bmatrix} \xrightarrow{\frac{1}{2}\begin{bmatrix} I & I \\ I & -I \end{bmatrix}} \begin{bmatrix} \frac{1}{2}(x_{even} + x_{odd}) \\ \frac{1}{2}(x_{even} - x_{odd}) \end{bmatrix}$$

$$T_a = \frac{1}{2}\begin{bmatrix} I & I \\ I & -I \end{bmatrix} \boxed{\text{split}}$$

$$\frac{1}{2}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \overbrace{\begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{cccc} e_0 & e_1 & e_2 & e_3 \end{array} \end{array}}^{\text{split}}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{array}{l} \leftarrow \text{scaling vector } (\tfrac{1}{2}) \\ \\ \leftarrow \text{wavelet vector } (\tfrac{1}{2}) \\ \\ \end{array}$$

Point of the video:

$$T_a = \begin{bmatrix} \mathcal{U} \quad m \times 2m \\ \hline \mathcal{V} \quad m \times 2m \end{bmatrix} \begin{matrix} \text{makes } s \\ \\ \text{makes } d \end{matrix}$$

$2m \times 2m$

$$T_a x = \begin{bmatrix} s \\ \hdashline d \end{bmatrix} \qquad \underbrace{\begin{bmatrix} \mathcal{U} \\ \hdashline \mathcal{V} \end{bmatrix}}_{T_a} x = \begin{bmatrix} \mathcal{U}x \\ \hdashline \mathcal{V}x \end{bmatrix}$$

$s = \mathcal{U}x$

$d = \mathcal{V}x$

$$T_a = \begin{bmatrix} \mathcal{U} \\ \hline \mathcal{V} \end{bmatrix} = D(U\text{'s } \& \ P\text{'s}) \ \boxed{\text{split}}$$

$$= \left[\begin{array}{c:c} & \\ \hline & \end{array} \right] \text{ split}$$

stuff built w/ powers of S

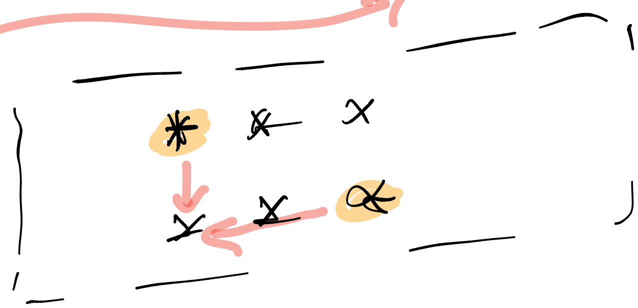
$I + S^{-1} \quad S + S^{-1}$

$3I$

these are circulant matrices!

$\mathcal{U}$ w/ rows shifted 1 down

$= \mathcal{U}$ w/ rows shifted 2 left



$S_m \mathcal{U} =$

$= \mathcal{U} S_{2m}^2$

$$\begin{bmatrix} S_m \mathcal{U} \\ S_m \mathcal{V} \end{bmatrix} = \begin{bmatrix} S_m & 0 \\ 0 & S_m \end{bmatrix} \begin{bmatrix} \mathcal{U} \\ \mathcal{V} \end{bmatrix}$$

$$= \begin{bmatrix} S_m & 0 \\ 0 & S_m \end{bmatrix} \underbrace{(D, P, \mathcal{U} \text{ stuff})}_{\text{built out of circulant things}} \boxed{\text{split}}$$

$$= \underbrace{(D, P, \mathcal{U} \text{ stuff}) \begin{bmatrix} S_m & 0 \\ 0 & S_m \end{bmatrix} \boxed{\text{split}}}$$

$$= (P, P, U \ast \mathrm{shift}) \boxed{\mathrm{split}}\ S^2_{2m}$$

$$T_a S^2_{2m}$$

$$= \begin{bmatrix} U \\ V \end{bmatrix} S^2_{2m} = \begin{bmatrix} U S^2_{2m} \\ V S^2_{2m} \end{bmatrix}$$

$$= \begin{bmatrix} S_m U \\ S_m V \end{bmatrix}$$