Given a wavelet transform with analysis matrix T_a, recall we may write

$$T_a x = \begin{bmatrix} \mathcal{U} \\ \hline \mathcal{V} \end{bmatrix} x = \begin{bmatrix} \mathcal{U}x \\ \hline \mathcal{V}x \end{bmatrix} = \begin{bmatrix} s \\ d \end{bmatrix}$$

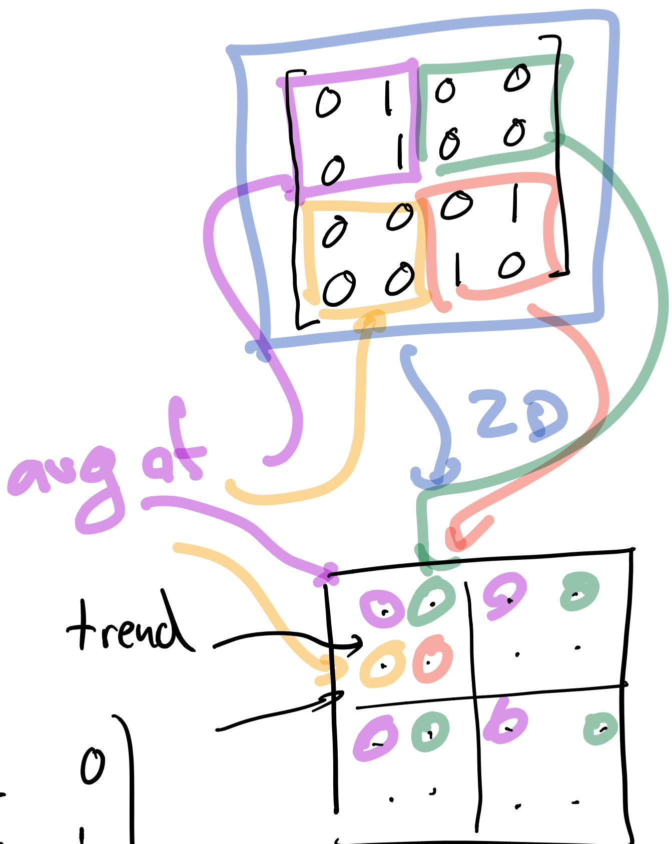
If we consider the 2-d wavelet transform

$$T_a z (T_a)^t = \begin{bmatrix} ss & sd \\ ds & dd \end{bmatrix}$$

how can we express the block entries in terms of $z, \mathcal{U}, \mathcal{V}, \mathcal{U}^t, \mathcal{V}^t$?

Apply the 2-d Haar Wavelet transform to the signals

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



$$\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

top: trends w/r/to columns

bottom: details w/r/to columns

left: trends w/r/to rows

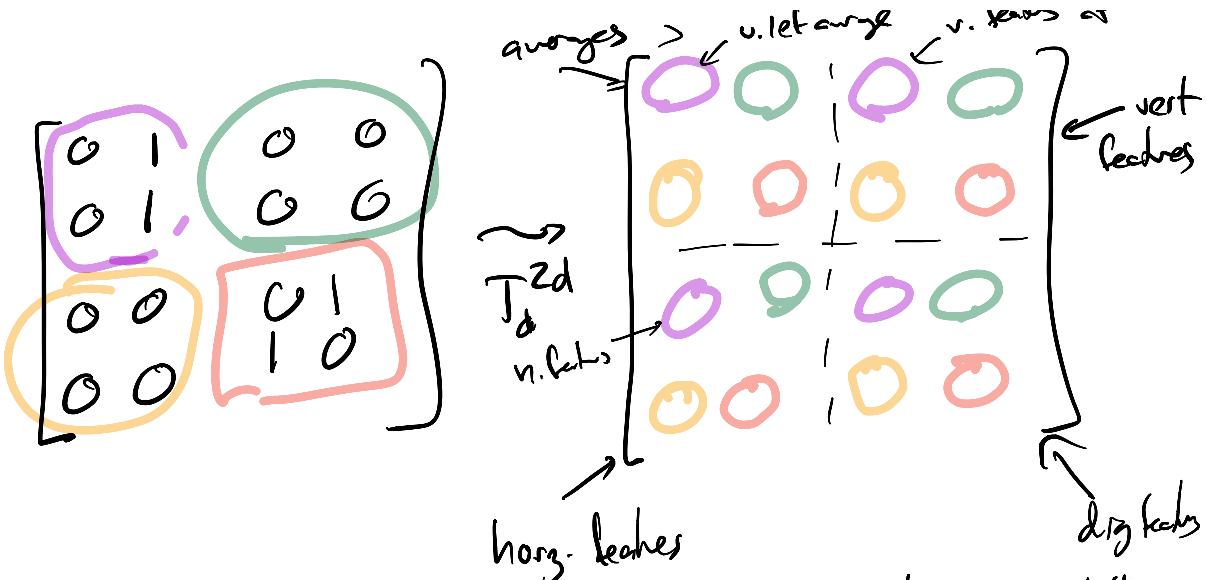
right: details w/r/to rows

ct rt	ct rd ← vert. features
cd rt	cd rd

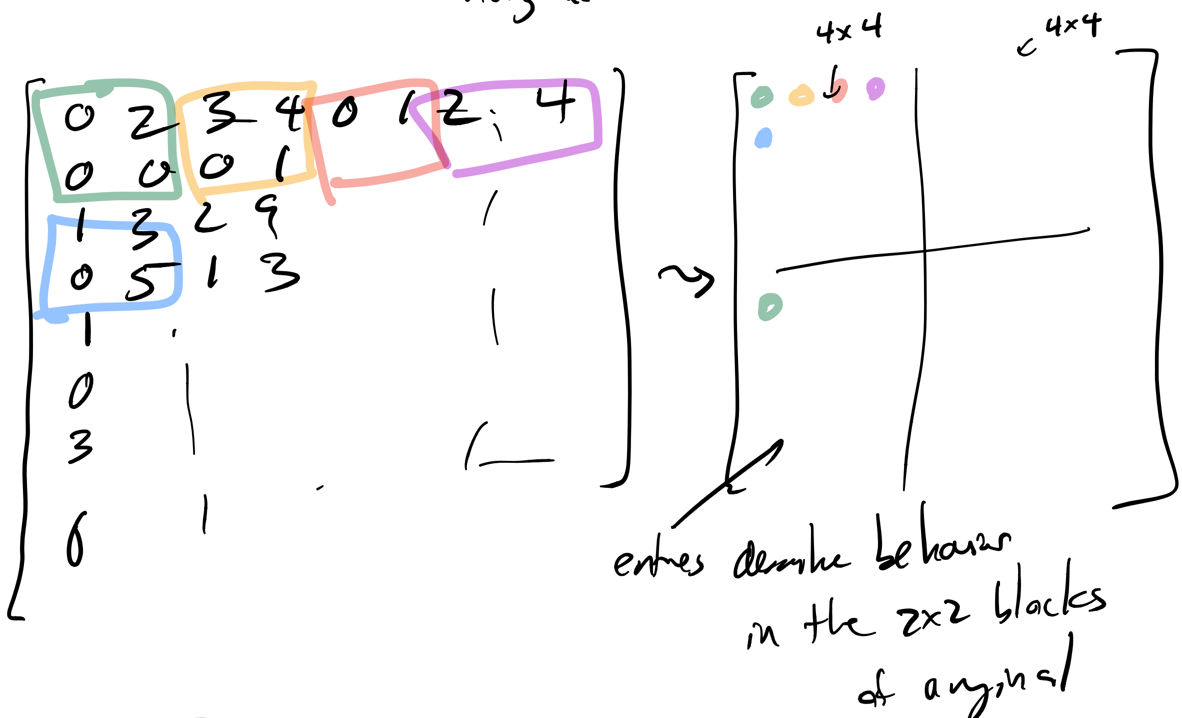
ex: $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \xrightarrow{\text{c.t.}} [0 \ \ 1] \xrightarrow{\text{r.d.}} [-\frac{1}{2}]$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{\text{c.t.}} [\frac{1}{2} \ \ \frac{1}{2}] \xrightarrow{\text{r.d.}} [0]$

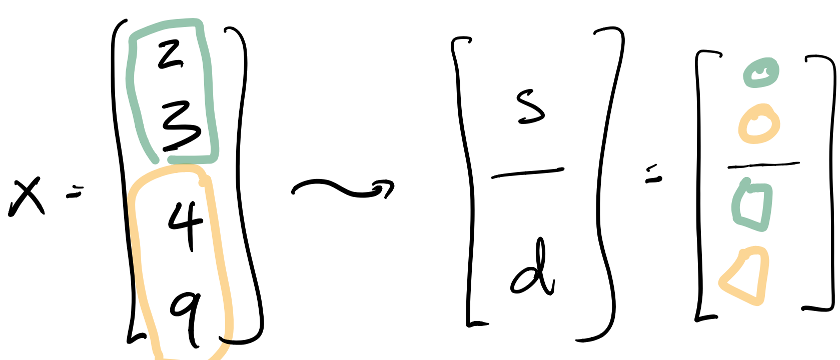
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \xrightarrow{\text{c.t.}} [0 \ \ 0] \longrightarrow [0]$



averages
vert features
horz. features
diag features



4x4
4x4
entries describe behavior in the 2x2 blocks of original



x =
s
d

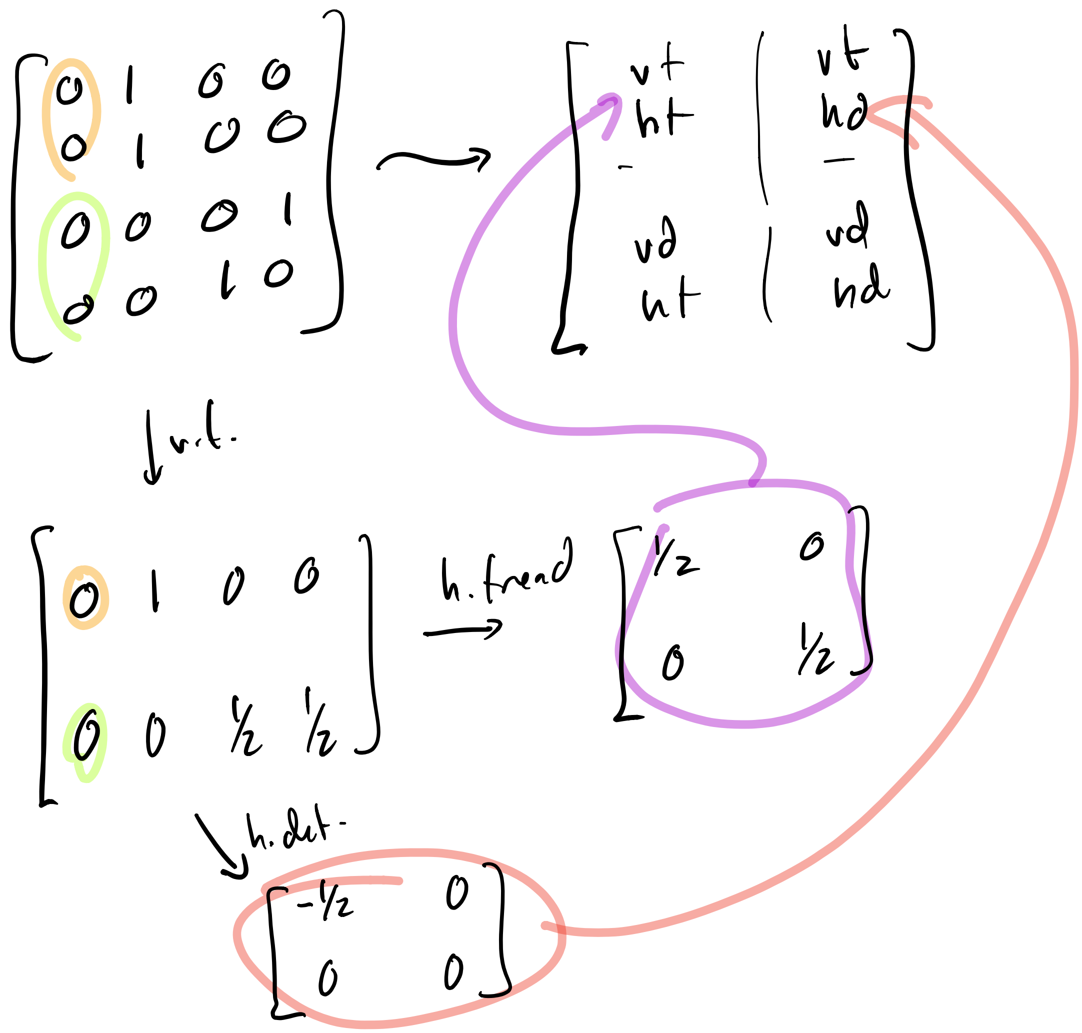
horiz features

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \xrightarrow{\text{r.t.}} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} \xrightarrow{\text{c.d}} [0]$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \xrightarrow{\text{r.d}} \begin{bmatrix} -1/2 \\ 1/2 \end{bmatrix} \xrightarrow{\text{c.d}} -\frac{1}{2}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{\text{r.d}} \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix} \xrightarrow{\text{c.d.}} \frac{1}{2}$$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \xrightarrow{\text{r.d}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \xrightarrow{\text{c.d.}} 0$$



$$T_a \times (T_a)^t = \begin{bmatrix} - & u & - \\ - & v & - \end{bmatrix} \times \begin{bmatrix} - & u & - \\ - & v & - \end{bmatrix}^t$$

↑ N×N matrix

$$= \begin{bmatrix} - & u & - \\ - & v & - \end{bmatrix} \times \left[u^t \,\middle|\, v^t \right]$$

$$= \begin{bmatrix} u_x \\ v_x \end{bmatrix} \left[u^t \,\middle|\, v^t \right]$$

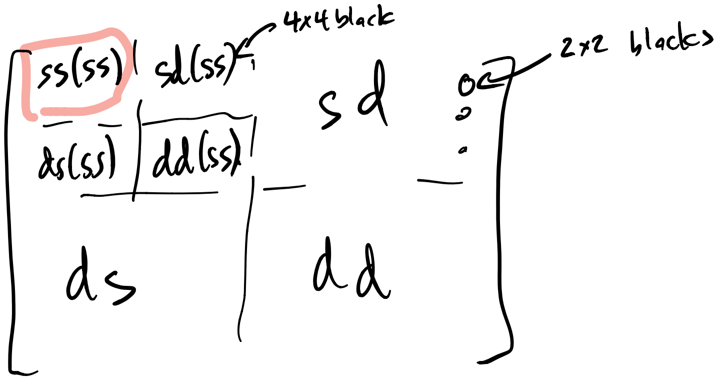
$$\left[\begin{array}{c} A \\ \hline B \end{array}\right] T = \left[\begin{array}{c} AT \\ \hline BT \end{array}\right]$$

$$T\left[\begin{array}{c|c} C & D \end{array}\right] = \left[\begin{array}{c|c} TC & TD \end{array}\right]$$

$$\left[\begin{array}{c} u_x \\ \hline v_x \end{array}\right]\left[\begin{array}{c|c} u^t & v^t \end{array}\right]$$

$$\left[\begin{array}{c} u_x \left[\begin{array}{c|c} u^t & v^t \end{array}\right] \\ \hline v_x \left[\begin{array}{c|c} u^t & v^t \end{array}\right] \end{array}\right] = \left[\begin{array}{c|c} u_x u^t & u_x v^t \\ \hline v_x u^t & v_x v^t \end{array}\right]$$

$$\left[\quad x \quad\right] \rightsquigarrow \left[\begin{array}{c|c} \boxed{ss} & sd \\ \hline ds & dd \end{array}\right]$$



ss(ss)
sd(ss)
4x4 block
ds(ss)
dd(ss)
sd
2x2 blocks
ds
dd