Haar

$$\boxed{\text{trend}} \quad s[k] = \tfrac{1}{2}x[2k] + \tfrac{1}{2}x[2k+1]$$

$$\boxed{\text{detail}} \quad d[k] = \tfrac{1}{2}x[2k] - \tfrac{1}{2}x[2k+1]$$

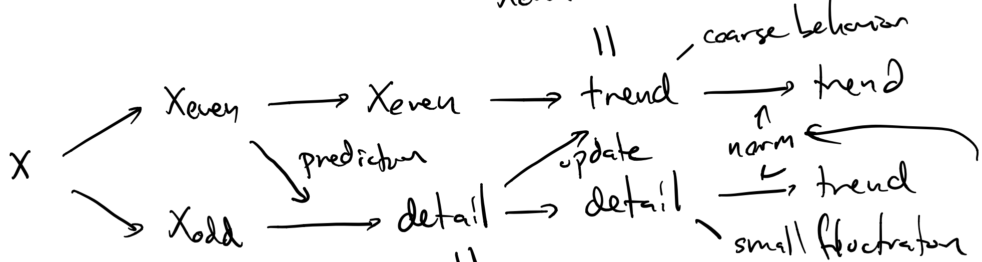
Assume x is periodic w/ period N

CDF(2,2)

$$x[2k] \sim s[k] = \left(-\tfrac{1}{8}x[2k-2] + \tfrac{1}{4}x[2k-1] + \tfrac{3}{4}x[2k] + \tfrac{1}{4}x[2k+1] - \tfrac{1}{8}x[2k+2]\right)\sqrt{2}$$

error at $x[2k+1]$ $\sim d[k] = \left(-\tfrac{1}{2}x[2k] + x[2k+1] - \tfrac{1}{2}x[2k+2]\right)\frac{1}{\sqrt{2}}$

$$X_{even} + \mathcal{U}\,\text{detail} = \begin{bmatrix} I_m & \mathcal{U} \\ 0 & I_m \end{bmatrix}\begin{bmatrix} X_{even} \\ X_{odd} \end{bmatrix}$$



$$\begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}$$

$$\text{detail} = X_{odd} \longrightarrow X_{odd} + \mathcal{P}\, X_{even}$$

$$\begin{bmatrix} I_m & 0 \\ \mathcal{P} & I_m \end{bmatrix} \cdot \begin{bmatrix} X_{even} \\ X_{odd} \end{bmatrix}$$

Prediction:

$$\begin{bmatrix} x_{even} \\ x_{odd} \end{bmatrix} \rightsquigarrow \begin{bmatrix} x_{even} \\ \underbrace{x_{odd} + \mathcal{P} x_{even}}_{\text{detail}} \end{bmatrix} = \boxed{\begin{bmatrix} I_m & 0 \\ \mathcal{P} & I_m \end{bmatrix}} \begin{bmatrix} x_{even} \\ x_{odd} \end{bmatrix}$$

$$d[k] = x[2k+1] - \frac{1}{2}\left(x[2k] + x[2k+1]\right)$$

$$= x_{odd}[k] - \frac{1}{2}\left(x_{even}[k] + x_{even}[k+1]\right)$$

$$= x_{odd}[k] - \frac{1}{2}\left(x_{even}[k] + (S^{-1} x_{even})[k]\right)$$

$$\boxed{d} = x_{odd} - \frac{1}{2} x_{even} - \frac{1}{2} S^{-1} x_{even}$$

$$= x_{odd} + \left(-\frac{1}{2} I_m - \frac{1}{2} S^{-1}\right) x_{even} = \boxed{x_{odd} + \mathcal{P} x_{even}}$$

$$\boxed{\mathcal{P} = -\frac{1}{2} I_m - \frac{1}{2} S^{-1}}$$

<u>Update</u>

$$\begin{bmatrix} x_{even} \\ d \end{bmatrix} \longrightarrow \begin{bmatrix} s \\ d \end{bmatrix}$$

$$d[k] = -\tfrac{1}{2} x[2k] + x[2k+1] - \tfrac{1}{2} x[2k+2]$$

$$x_{even}[k] = x[2k]$$

$$s = -\tfrac{1}{8} x[2k-2] + \tfrac{1}{4} x[2k-1] + \tfrac{3}{4} x[2k] + \tfrac{1}{4} x[2k+1] - \tfrac{1}{8} x[2k+2]$$

	$2k-2$	$2k-1$	$2k$	$2k+1$	$2k+2$
$s[k]$	$-\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{8}$
$d[k]$	0	0	$-\frac{1}{2}$	1	$-\frac{1}{2}$
$x_{even}[k]$	0	0	1	0	0
$Sd[k]$	$-\frac{1}{2}$	1	$-\frac{1}{2}$	0	0

$$s[k] = x_{even}[k] + \tfrac{1}{4} d[k] + \tfrac{1}{4} d[k-1]$$

$$= x_{even}[k] + \tfrac{1}{4} d[k] + \tfrac{1}{4} (Sd)[k]$$

$$s = x_{even} + \tfrac{1}{4} d + \tfrac{1}{4} Sd = x_{even} + \left(\tfrac{1}{4} I_m + \tfrac{1}{4} S\right) d$$

$$\begin{bmatrix} X_{even} \\ d \end{bmatrix} \rightsquigarrow \begin{bmatrix} s \\ d \end{bmatrix} = \begin{bmatrix} I_m & \mathcal{U} \\ 0 & I_m \end{bmatrix} \begin{bmatrix} X_{even} \\ d \end{bmatrix}$$

$$\mathcal{U} = \frac{1}{4} I_m + \frac{1}{4} S$$

normalization: $D = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 1/\sqrt{2} \end{bmatrix}$.

$$T_a = D\,U\,P\,[\text{split}]$$

$$P = \begin{bmatrix} I_m & 0 \\ \mathcal{P} & I_m \end{bmatrix} \qquad U = \begin{bmatrix} I_m & \mathcal{U} \\ 0 & I_m \end{bmatrix}$$

Synthesis & Wavelet Bases

Liner Algebra Cheap tricks

$$M_{2m} \qquad \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E & F \\ G & H \end{bmatrix} = \begin{bmatrix} AE + BG & \overset{\neq FA}{AF + BH} \\ CE + DG & \boxed{CF + DH} \end{bmatrix}$$

$$A, B, C, D, E, F, G, H \in M_m(\mathbb{C})$$

$$\begin{bmatrix} I_m & 0 \\ A & I_m \end{bmatrix}^{-1} = \begin{bmatrix} I_m & 0 \\ -A & I_m \end{bmatrix}$$

$$\begin{bmatrix} I_m & B \\ 0 & I_m \end{bmatrix}^{-1} = \begin{bmatrix} I_m & -B \\ 0 & I_m \end{bmatrix}$$

$$T_a = DUP\,\boxed{\text{split}} \qquad T_s = T_a^{-1} = \boxed{\text{split}}^{-1} P^{-1} U^{-1} D^{-1}$$

$$D = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} \quad D^{-1} = \begin{bmatrix} d_1^{-1} & 0 \\ 0 & d_2^{-1} \end{bmatrix}$$

$$U = \begin{bmatrix} I_m & \mathcal{U} \\ 0 & I_m \end{bmatrix} \quad U^{-1} = \begin{bmatrix} I_m & -\mathcal{U} \\ 0 & I_m \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} I_m & 0 \\ -\beta & I_m \end{bmatrix}$$

$$\mathcal{U} = \frac{1}{4} I_m + \frac{1}{4} S$$

$$U = \begin{bmatrix} I_m & \left(\frac{1}{4} I_m + \frac{1}{4} S\right) \\ 0 & I_m \end{bmatrix} \quad U^{-1} = \begin{bmatrix} I_m & -\frac{1}{4} I_m - \frac{1}{4} S \\ 0 & I_m \end{bmatrix}$$

write down matrix form for

T_s