The Daub4 Wavelet transform

$$\begin{tikzcd} x \arrow[r, squiggly] \arrow[rr, bend right, "T_a"'] & \begin{bmatrix} x_{even} \\ x_{odd} \end{bmatrix} \arrow[r, squiggly] & \begin{bmatrix} s \\ d \end{bmatrix} \end{tikzcd}$$

$$s[k] = \left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)\Bigg(\left(\frac{\sqrt{3}+2}{4}\right)x[2k] + \left(\frac{\sqrt{3}}{4}\right)x[2k+2] + \left(\frac{3-\sqrt{3}}{4}\right)x[2k+1] - \frac{1}{4}x[2k+3]\Bigg)$$

$$d[k] = \left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)\Bigg(\left(\frac{2-\sqrt{3}}{4}\right)x[2k-2] - \left(\frac{\sqrt{3}}{4}\right)x[2k] + \left(\frac{2\sqrt{3}-3}{4}\right)x[2k-1] + \frac{1}{4}x[2k+1]\Bigg)$$

	$2k-2$	$2k-1$	$2k$	$2k+1$	$2k+2$	$2k+3$
$s^{(2)}$			$\frac{\sqrt{3}+2}{4}$	$\frac{3-\sqrt{3}}{4}$	$\frac{\sqrt{3}}{4}$	$-\frac{1}{4}$
$d^{(1)}$	$\frac{2-\sqrt{3}}{4}$	$\frac{2\sqrt{3}-3}{4}$	$-\frac{\sqrt{3}}{4}$	$\frac{1}{4}$		
$s^{(1)}$			1	$\sqrt{3}$		
"predict"	$\frac{\sqrt{3}-2}{4}$	$\frac{3-2\sqrt{3}}{4}$	$\frac{\sqrt{3}}{4}$	$\frac{3}{4}$		

$$s^{(1)} = x_{even} + \sqrt{3}\, x_{odd}$$

$$\text{"predict"} = \left(\frac{\sqrt{3}-2}{4}\right) S s^{(1)} + \left(\frac{\sqrt{3}}{4}\right) s^{(1)}$$

$$\boxed{d^{(1)} = x_{odd} - \text{"predict"}}$$

$$s^{(2)} = s^{(1)} - S^{-1} d^{(1)}$$

$\boxed{\text{Procedure}}$

$$x \longrightarrow \begin{bmatrix} x_{even} \\ x_{odd} \end{bmatrix} \xrightarrow{U_1} \begin{bmatrix} s^{(1)} \\ x_{odd} \end{bmatrix}$$

$$s^{(1)} = x_{even} + \sqrt{3}\, x_{odd}$$

$$= \begin{bmatrix} I & \sqrt{3} I \end{bmatrix} \begin{bmatrix} x_{even} \\ x_{odd} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} I & \sqrt{3} I \\ 0 & I \end{bmatrix}}_{U_1} \begin{bmatrix} x_{even} \\ x_{odd} \end{bmatrix} = \begin{bmatrix} s^{(1)} \\ x_{odd} \end{bmatrix}$$

$$\begin{bmatrix} s^{(1)} \\ x_{odd} \end{bmatrix} \xrightarrow{P_1} \begin{bmatrix} s^{(1)} \\ d^{(1)} \end{bmatrix}$$

$$d^{(1)} = x_{odd} - \left(\frac{\sqrt{3}-2}{4}\right) S s^{(1)} + \left(\frac{\sqrt{3}}{4}\right) s^{(1)}$$

$$= x_{odd} - \left(\left(\frac{\sqrt{3}-2}{4}\right) S + \left(\frac{\sqrt{3}}{4}\right) I\right) s^{(1)}$$

$$= \begin{bmatrix} \left(-\left(\frac{\sqrt{3}-2}{4}\right) S - \frac{\sqrt{3}}{4} I\right) & I \end{bmatrix} \begin{bmatrix} s^{(1)} \\ x_{odd} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} I & 0 \\ -\left(\frac{\sqrt{3}-2}{4}\right) S - \frac{\sqrt{3}}{4} I & I \end{bmatrix}}_{P_1} \begin{bmatrix} s^{(1)} \\ x_{odd} \end{bmatrix} = \begin{bmatrix} s^{(1)} \\ d^{(1)} \end{bmatrix}$$

$$s^{(2)} = s^{(1)} - S^{-1} d^{(1)} = \begin{bmatrix} I & -S^{-1} \end{bmatrix} \begin{bmatrix} s^{(1)} \\ d^{(1)} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} I & -S^{-1} \\ 0 & I \end{bmatrix}}_{U_2} \begin{bmatrix} s^{(1)} \\ d^{(1)} \end{bmatrix} = \begin{bmatrix} s^{(2)} \\ d^{(1)} \end{bmatrix}$$

$$D = \begin{bmatrix} \frac{\sqrt{3}-1}{2} & 0 \\ 0 & \frac{\sqrt{3}+1}{2} \end{bmatrix}$$

$$D \begin{bmatrix} s^{(2)} \\ d^{(1)} \end{bmatrix} = \begin{bmatrix} s \\ d \end{bmatrix}$$

$$\underbrace{D U_2 P_1 U_1 \boxed{\text{split}}}_{T_a} x = \begin{bmatrix} s \\ d \end{bmatrix}$$

$$D U_2 P_1 U_1 = \begin{bmatrix} \alpha I + \gamma S^{-1} & \beta I + \delta S^{-1} \\ -\beta I - \delta S & \alpha I + \gamma S \end{bmatrix}$$

$$\alpha = \frac{1+\sqrt{3}}{4\sqrt{2}} \quad \beta = \frac{3+\sqrt{3}}{4\sqrt{2}} \quad \gamma = \frac{3-\sqrt{3}}{4\sqrt{2}} \quad \delta = \frac{1-\sqrt{3}}{4\sqrt{2}}$$

$$DU_2P_1U_1 = \begin{bmatrix} \left(\frac{1+\sqrt{3}}{4\sqrt{2}} + \frac{3-\sqrt{3}}{4\sqrt{2}} S^{-1}\right) & \left(\frac{3+\sqrt{3}}{4\sqrt{2}} + \frac{1-\sqrt{3}}{4\sqrt{2}} S^{-1}\right) \\ \left(-\frac{3+\sqrt{3}}{4\sqrt{2}} - \frac{1-\sqrt{3}}{4\sqrt{2}} S\right) & \left(\frac{1+\sqrt{3}}{4\sqrt{2}} + \frac{3-\sqrt{3}}{4\sqrt{2}} S\right) \end{bmatrix}$$