$$T_a x = \begin{bmatrix} s \\ d \end{bmatrix}$$

$$T_a = \left[\begin{array}{c} \mathcal{U} \\ \hline \mathcal{V} \end{array}\right]$$

$m \times 2m$

$2m \times 2m$

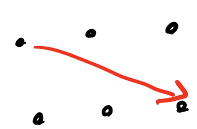
$$s = \mathcal{U}x$$
$$d = \mathcal{V}x$$

$$\left[\begin{array}{c} \mathcal{U} \\ \hline \mathcal{V} \end{array}\right] \begin{bmatrix} x \end{bmatrix} = \left[\begin{array}{c} \mathcal{U}x \\ \hline \mathcal{V}x \end{array}\right] = \begin{bmatrix} s \\ d \end{bmatrix}$$

u_0 = scaling vector = top row of $\mathcal{U}$

v_0 = wavelet vector = top row of $\mathcal{V}$

<u>Symmetry</u>:



$$\mathcal{U} = \begin{bmatrix} a & b & c & d & e & f & g \\ f & g & a & b & c & d & e \\ d & e & f & g & a & b & c \end{bmatrix}$$

updates & prediction matrices

$$\begin{bmatrix} I & * \\ 0 & I \end{bmatrix} \qquad \begin{bmatrix} I & 0 \\ * & I \end{bmatrix}$$

$*$ = linear combinations of shift matrices

$$a_{-1}S^{-1} + a_0 \underset{\uparrow \atop S^0}{I} + a_1 S + a_2 S^2$$

"Laurent polynomial in S"

$$D = \begin{bmatrix} \lambda I & 0 \\ 0 & \mu I \end{bmatrix}$$

$$T_a = \underbrace{D\left(\text{a bunch of } U\text{'s } \& \ P\text{'s}\right)} \quad \boxed{\text{split}}$$

$$T_a = \begin{bmatrix} p_{00}(S) & p_{01}(S) \\ p_{10}(S) & p_{11}(S) \end{bmatrix} \quad \boxed{\text{split}}$$

$p_{ij}(S)$ = Laurent polys in S

$$T_a = \begin{bmatrix} p_{00}(s) & p_{01}(s) \\ p_{10}(s) & p_{11}(s) \end{bmatrix} \boxed{\text{split}} = \begin{bmatrix} \mathcal{U} \\ \hline \mathcal{V} \end{bmatrix}$$

symmetry property of $\mathcal{U}$ & V:

shift rows down 1 = shift columns left 2

$$\begin{bmatrix} a & b & c & d & e \\ d & e & a & b & c \end{bmatrix} \xrightarrow{\downarrow 1} \begin{bmatrix} \text{stuff} \\ a \; b \; c \; d \; e \end{bmatrix}$$

$$\begin{bmatrix} a & b & c & d & e \\ d & e & a & b & c \end{bmatrix} \xrightarrow{\leftarrow 2} \begin{bmatrix} c & d & e & a & b \\ a & b & c & d & e \end{bmatrix}$$

$$S_m \begin{bmatrix} x_0 \\ \vdots \\ x_{m-1} \end{bmatrix} = \begin{bmatrix} x_{m-1} \\ x_0 \\ \vdots \\ x_{m-2} \end{bmatrix}$$

$$[y_0 \; y_1 \; \cdots \; y_{2m-1}] \, S_{2m} = [y_0 \; \cdots \; y_{2m-1}] \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & & & & \vdots \\ 0 & 1 & & & & \vdots \\ \vdots & \vdots & & \ddots & & \vdots \\ 0 & 0 & & & 1 & 0 \end{bmatrix}$$

$$= [y_1 \; y_2 \; \cdots \; y_{2m-1} \; y_0]$$

$S_m \mathcal{U} = \mathcal{U}$ with rows shifted down 1

$\mathcal{U} S_{2m}^2 = \mathcal{U}$ with columns shifted left 2

<u>want:</u>

$$S_m \mathcal{U} = \mathcal{U} S_{2m}^2$$
$$S_m \mathcal{V} = \mathcal{V} S_{2m}^2$$

$$\begin{bmatrix} S_m \mathcal{U} \\ \hline S_m \mathcal{V} \end{bmatrix} = \begin{bmatrix} S_m & 0 \\ 0 & S_m \end{bmatrix} \begin{bmatrix} \mathcal{U} \\ \hline \mathcal{V} \end{bmatrix}$$

$$= \begin{bmatrix} S_m & 0 \\ 0 & S_m \end{bmatrix} \begin{bmatrix} p_{00}(S) & p_{01}(S) \\ p_{10}(S) & p_{11}(S) \end{bmatrix} \boxed{\text{split}}$$

$$= \begin{bmatrix} S_m p_{00}(S_m) & S_m p_{01}(S_m) \\ S_m p_{10}(S_m) & S_m p_{11}(S_m) \end{bmatrix} \boxed{\text{split}}$$

$$= \begin{bmatrix} p_{00}(S) S_m & p_{01}(S) S_m \\ p_{10}(S) S_m & p_{11}(S) S_m \end{bmatrix} \boxed{\text{split}}$$

$$\begin{bmatrix} S_m \mathcal{U} \\ \hline S_m \mathcal{V} \end{bmatrix} = \begin{bmatrix} p_{00}(S) & p_{01}(S) \\ p_{10}(S) & p_{11}(S) \end{bmatrix} \begin{bmatrix} S_m & 0 \\ 0 & S_m \end{bmatrix} \boxed{\text{split}}$$

$$= \begin{bmatrix} \text{''} \end{bmatrix} [\text{split}] \, S_{2m}^2$$

$$\begin{bmatrix} S_m & 0 \\ 0 & S_m \end{bmatrix} \boxed{\text{split}} = \boxed{\text{split}}\, S_{2m}^2$$

$$\begin{bmatrix} S_m u \\ \hline S_m v \end{bmatrix} = \begin{bmatrix} u \\ \hline v \end{bmatrix} S_{2m}^2 = \begin{bmatrix} u S_{2m}^2 \\ \hline v S_{2m}^2 \end{bmatrix}$$

$$\begin{bmatrix} S_m & 0 \\ 0 & S_m \end{bmatrix} \boxed{\text{split}} = \boxed{\text{split}}\, S_{2m}^2 \quad ?$$

$$\begin{bmatrix} S_m & 0 \\ 0 & S_m \end{bmatrix} \begin{bmatrix} x_0 \\ x_2 \\ \vdots \\ x_{2m-2} \\ \hdashline x_1 \\ x_3 \\ \vdots \\ x_{2m-1} \end{bmatrix} = \begin{bmatrix} x_{2m-2} \\ x_0 \\ x_2 \\ \vdots \\ x_{2m-4} \\ \hdashline x_{2m-1} \\ x_1 \\ x_3 \\ \vdots \\ x_{2m-3} \end{bmatrix}$$

$$\boxed{\text{split}}\, S_{2m}^2 \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{2m-1} \end{bmatrix} = \boxed{\text{split}} \begin{bmatrix} x_{2m-2} \\ x_{2m-1} \\ x_0 \\ x_1 \\ \vdots \\ x_{2m-3} \end{bmatrix} \quad //$$