Applied Algebra Practice Sheet for Exam 2

This sheet is not meant to be exhaustive, but rather as a supplement to the problems from the homework since the last exam.

1.

Suppose T_a is the analysis matrix in the case $N = 4$, for a wavelet transform with scaling vector $(1, 2, 0, 0)$ and wavelet vector of $(2, -3, 1, 0)$. Compute $T_a(1, 1, 1, 1)$ and $T_a(1, -1, 0, 0)$.

2.

Suppose we have a wavelet transform given by $x \mapsto (s, d)$ where

$$d[k] = x[2k + 1] - x[2k] - 2x[2k + 2]$$

and

$$s[k] = x[2k] + d[k] + 3d[k - 1].$$

Find block matrices P, U such that $T_a = UP$ $\boxed{\text{split}}$.

3.

For the wavelet transform in the previous problem, find the scaling and wavelet vectors in the case $N = 4$.

4.

Recall the two-scale Haar Transform $x \mapsto (s_1, s_2, d_2)$. This is given by first performing the Haar transform $x \mapsto (s_1, d_1)$, then performing a second Haar transform $s_1 \mapsto (s_2, d_2)$ on the first trend, and collecting this all in the vector (s_1, s_2, d_2). Give a description, in block form, for the matrix which gives this linear transformation $x \mapsto (s_1, s_2, d_2)$.

Hint: you may need more than 4 blocks!

5.

Consider the following procedures, where T_a, T_s are the analysis and synthesis matrices for the Haar transform.

- apply T_a, look at the coordinates of the resulting vector and set to 0 all coordinates which are sufficiently small. Then apply T_s to the resulting vector
- apply T_a to get s, d, replace d with 0, then apply T_s.
- apply T_a to get s, d, replace s with 0, then apply T_s.

Consider the following goals we might have:

- remove noise from a signal
- find jumps in signal
- compress signal

Which of the above operations would be potentially useful for these goals?
