Lecture 11: Applications of the chain rule

Tuesday, February 7, 2017 12:33 PM

1. $\frac{d}{dx} e^{xe^x} = e^{xe^x} \frac{d}{dx} xe^x = e^{xe^x}\left(\left(\frac{d}{dx}x\right)e^x + x\frac{d}{dx}e^x\right)$

$= e^{xe^x}\left(1 \cdot e^x + xe^x\right)$

$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

2. $\frac{d}{dx} \sin(x^2) e^x = \left(\frac{d}{dx}\sin(x^2)\right)(e^x) + \sin x^2 \left(\frac{d}{dx} e^x\right)$

$= \left(\cos(x^2) \cdot \frac{d}{dx}x^2\right) e^x + (\sin x^2) e^x$

$e^x 2x \cos x^2 + e^x \sin x^2$

3. $\frac{d}{dx} \sin(\cos(e^{e^x})) = \cos(\cos(e^{e^x})) \cdot \frac{d}{dx} \cos(e^{e^x})$

$= \cos(\cos(e^{e^x})) \cdot (-\sin(e^{e^x})) \cdot \frac{d}{dx} e^{e^x}$

$= \cos(\cos(e^{e^x})) \cdot (-\sin(e^{e^x})) \cdot e^{e^x} \cdot e^x$

4. $\frac{d}{dx} \sin(\sin(\sin(\sin e^x)))$

$= \cos(\sin(\sin(\sin e^x))) \cdot \underbrace{\frac{d}{dx}\sin(\sin(\sin e^x))}$

$\cos(\sin(\sin(e^x))) \cdot \underbrace{\frac{d}{dx}\sin(\sin e^x)}$

$\cos(\sin e^x) \cdot \underbrace{\frac{d}{dx}\sin e^x}$

$\cos e^x \cdot \frac{d}{dx} e^x$

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5. $\frac{d}{dx} 3x^2 - 2\sqrt{1-x} + e^{2x}$

$\frac{d}{dx} 3x^2 - 2(1-x)^{1/2} + e^{2x}$

$= \frac{d}{dx}(3x^2) - 2\frac{d}{dx}(1-x)^{1/2} + \frac{d}{dx} e^{2x}$

$6x - 2 \cdot \frac{1}{2}(1-x)^{-1/2} \cdot \frac{d}{dx}(1-x) + e^{2x} \cdot \frac{d}{dx} 2x$

$6x - (1-x)^{-1/2}(-1) + 2e^{2x}$

$\frac{d}{dx}(1+\sin x)^6 = 6(1+\sin x)^5 \cdot \frac{d}{dx}(1+\sin x)$

$= 6(1+\sin x)^5 \cos x$

outer: x^6
inner: $1+\sin x$

$\frac{d}{dx} \sin(1-x^2) = \cos(1-x^2) \cdot (-2x)$

outer: $\sin x$
inner: $1-x^2$

$\frac{d}{dx} \cos(1-\sqrt{2+e^x}) = -\sin(1-\sqrt{2+e^x}) \cdot \frac{d}{dx}(1-\sqrt{2+e^x})$

outer: cos

$(1-\sqrt{2+e^x})(-\frac{d}{dx}(2+e^x)^{1/2})$

dx

outer = cos
inner $1-\sqrt{2+e^x}$

$= -\sin(1-\sqrt{2+e^x})\left(-\frac{d}{dx}(2+e^x)^{1/2}\right)$

$= -\sin(1-\sqrt{2+e^x})\left(-\frac{1}{2}(2+e^x)^{-1/2}\underbrace{\frac{d}{dx}(2+e^x)}_{e^x}\right)$

$\frac{d}{dx}1-\sqrt{2+e^x} = \frac{d}{dx}1 - \frac{d}{dx}(2+e^x)^{1/2}$

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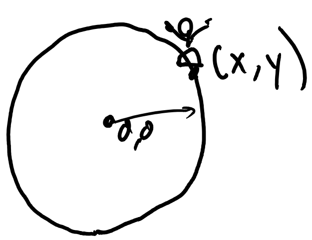
ex: $PV = nRT \longrightarrow T = \frac{1}{nR}PV$

$$\frac{dT}{dt} = \frac{1}{nR}\cdot\frac{d}{dt}PV = \frac{1}{nR}\left(\frac{dP}{dt}V + P\cdot\frac{dV}{dt}\right) \dots$$

ex:



100 ft radius

$x^2+y^2 = 100^2$

solve for $\frac{dx}{dt}$.

$x = x(t)$
$y = y(t)$

$$\frac{d}{dt}(x^2+y^2) = \frac{d}{dt}(100^2)$$

$$\frac{d}{dt}x^2 + \frac{d}{dt}y^2 = 0 \longrightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$\frac{d}{dt}x(t)^2 = 2(x(t))\cdot\frac{dx}{dt}$ $\quad \frac{dx}{dt} = -2y\frac{dy}{dt}$

$$\frac{d}{dt}x(t)^2 = 2(x(t)) \cdot \frac{dx}{dt}$$

$$\frac{d}{dt}y(t)^2 = 2y(t) \cdot \frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{-2y\frac{dy}{dt}}{2x}$$

$$= -\frac{y}{x}\frac{dy}{dt}$$

<u>Idea:</u> Everything is a function of anything.

$$\frac{d}{dx}x = 1 \qquad \frac{d}{dx}y = \frac{dy}{dx}$$

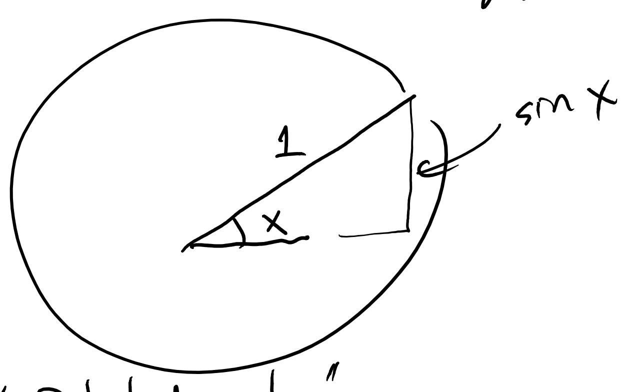
$$\frac{d}{dx}y^2 = 2y\frac{dy}{dx} \qquad \frac{d}{dt}y^2 = 2y\frac{dy}{dt}$$

$$\frac{d}{dt}xy = x\frac{dy}{dt} + y\frac{dx}{dt} \qquad \frac{d}{dx}xy =$$

$$\frac{d}{dy}xy =$$

$$\frac{d}{dt}\sin x = \cos x \cdot \frac{d}{dt}x$$

$$= \cos x \frac{dx}{dt}$$



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