Lecture 12: Hard problems

Thursday, February 9, 2017 1:52 PM

How do you solve

$$x^2 - 3x + 1 = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

known since ancient times

$$x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$

How do you solve

$$x^3 - 3x^2 + 2x + 4 = 0$$

$$x^2(x-3) = -2(x+2)$$

method: try to get lucky.

use the cubic formula! Thanks, Cardano.

known since 1545

How do you solve

$$x^4 - 3x^3 + 2x^2 + 7x - 52 = 0$$

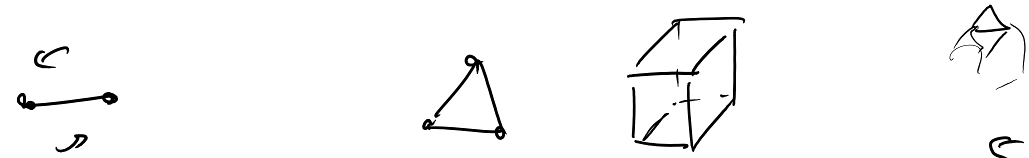
again, Cardano is victorious. (stolen) 1545

Degree 5?

$$x^5 \quad ax^4 - 3x^3 + 2x^2 + 7x - 52 = 0$$

none exist. ≈ 1830 Evarise Galois

no way to express the solutions using standard mathematical symbols.



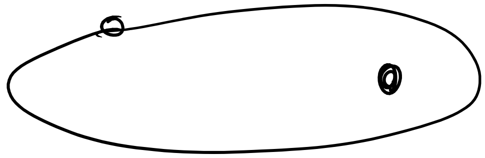
20 sided

2 3 4 5

S_5

not solvable.

Planets travel in elliptical orbits (until Einstein)



natural question: arclength of an ellipse?

no way to express the arclength in terms of "familiar functions"

instead: invent "elliptical functions"

$\sqrt{5} = ?$

roughly 2.2 ?

$(2.2)^2 = (2)^2(1.1)^2 = 4(1.21) = 4.84$ too small

2.3? $(2.3)^2 = 5.29$ too big.

$(2.25)^2 = 5.0625$

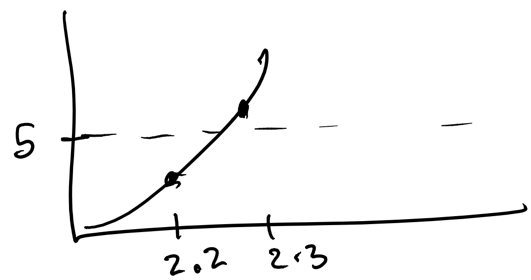
$$\begin{array}{r} 23 \\ \underline{23} \\ 69 \\ 46\cdot \end{array} \qquad \begin{array}{r} 225 \\ \underline{2} \\ 550 \end{array} \qquad \begin{array}{r} 225 \\ \underline{5} \\ 1125 \end{array} \qquad \begin{array}{r} 225 \\ \underline{225} \\ 1125 \\ 450\cdot \\ \underline{450\cdot\cdot} \\ 50625 \end{array}$$

<u>Method</u>: Intermediate Value theorem

$f(x) = x^2$ $f(2.2) = 4.84 < 5$

$f(2.3) = 5.29 > 5$

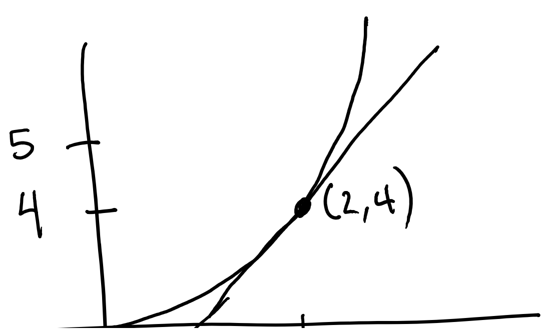
$IVT \Rightarrow f(c) = 5$ for some $2.2 < c < 2.3$



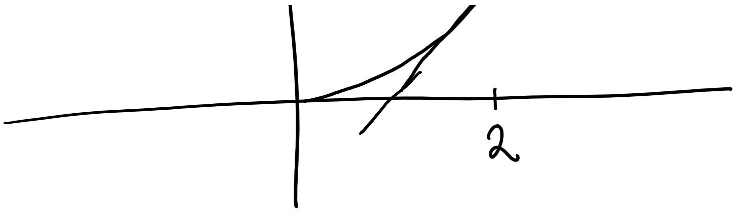
"midpoint method"

<u>Alternately</u>:

Solve $f(x) = 5$ where $f(x) = x^2$ $x = 2$ guess.



Philosophy:
tangent line is very close to $f(x)$ if x is close to 2.



is close to 2.

Instead of solving $f(x) = 5$, we let $L(x)$ = eqn of tangent line at $x=2$ and solve $L(x) = 5$ instead.

$L(x)$ line slope $= f'(2)$ $\quad f'(x) = 2x \quad f'(2) = 2(2) = 4$

$= 4$

$= 4x - 4$

passes through pt: $(2,4)$

$y - y_1 = m(x - x_1) \rightsquigarrow y - 4 = 4(x-2)$

$y = 4x - 8 + 4 = 4x - 4$

Solve:

$L(x) = 5$

$4x - 4 = 5$

$x = \frac{9}{4} \approx \sqrt{5}$? $\quad \left(\frac{9}{4}\right)^2 = \frac{81}{16} = 5\frac{1}{16}$

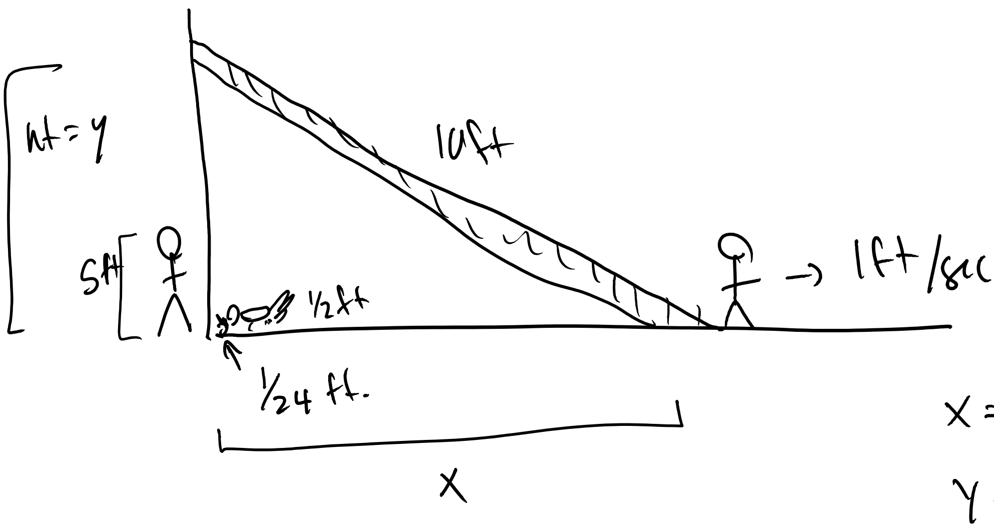
from here, could use this as guess, repeat.

"Newton's Method"

General Idea: "Linear Approximation"

tangent lines are good approximations for most functions

used by computers, calculators, everybody.



how fast are the beings under ladder getting hit?

$x = x(t) = f(t)$

$y = y(t) = g(t)$

$\frac{dx}{dt} = 1\ \text{ft/sec} = 1$

$\frac{dy}{dt} = \text{want}$

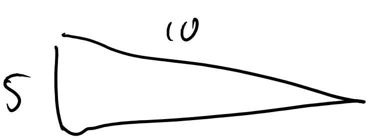
$x^2 + y^2 = 10^2$

$f(t)^2 + g(t)^2 = 10^2$

$\frac{d}{dt}(x^2+y^2) = \frac{d}{dt}(10^2)$

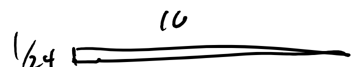
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \longrightarrow \frac{dy}{dt} = \frac{-2x\frac{dx}{dt}}{2y}$

need to know $x, y, \frac{dx}{dt}$. $\longleftarrow$ $\boxed{\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}}$



(human)

squirrel



beetle

(human)

$x = \sqrt{10^2 - 5^2} = \sqrt{75} = 5\sqrt{3} \approx 8$

$y = 5$

$\frac{dx}{dt} = 1$

$-\frac{8}{5}\cdot 1 = -\frac{8}{5}$

squirrel

$x \approx 10$

$y = 1/2$

$\frac{dx}{dt} = 1$

beetle

$x \approx 10$

$y = 1/24$

$\frac{dx}{dt} = 1$

$-\frac{10}{\ } = -240$

dt at

$-\frac{8}{5}\cdot 1 = -\frac{8}{5}$

$-\frac{10}{(1/2)} = -20$

$-\frac{1}{(1/24)} = -240$

$$\lim_{x\to 0^+} -\frac{x}{y}\frac{dx}{dt} = \lim_{y\to 0^+} -\frac{10}{y} = -\infty$$