Warm ups

1. $\frac{d}{dx} x^2 \cos 2x$

2. $\frac{d}{dx} \frac{\cos(e^x)}{1+x}$

3. if $x = e^y$ find $\frac{dy}{dx}$ (in terms of x)

1. $\lim_{x \to 9} \frac{(x-3)(\sqrt{x}-3)}{x-9}$

2. $\lim_{x \to 4} \frac{x - \sqrt{3x+4}}{x-4}$

3. $\boxed{\lim_{x \to 0} \frac{1-\cos 2x}{x^2}}$

4. $\lim_{x \to 0} \frac{\sqrt{1+\sin x} - 1}{\sqrt{x}}$

5. $\lim_{x \to 1} \frac{\sin(1-x)}{x-1}$

2. $\lim_{x \to 4} \frac{x - \sqrt{3x+4}}{x-4} = \lim_{x \to 4} \left(\frac{x - \sqrt{3x+4}}{x-4}\right)\left(\frac{x+\sqrt{3x+4}}{x+\sqrt{3x+4}}\right)$

$x^2 - (3x+4)$

$= \lim \frac{x^2-3x-4}{\dots}$

$$= \lim_{x\to 4} \frac{x^2-(3x+4)}{(x-4)(x+\sqrt{3x+4})} = \lim_{x\to 4} \frac{x^2-3x-4}{(x-4)(x+\sqrt{3x+4})}$$

$$= \lim_{x\to 4} \frac{(x+1)\cancel{(x-4)}}{\cancel{(x-4)}(x+\sqrt{3x+4})} = \lim_{x\to 4} \frac{(x+1)}{(x+\sqrt{3x+4})} = \frac{4+1}{4+\sqrt{3\cdot 4+4}}$$

$$= \frac{5}{4+\sqrt{16}} = \frac{5}{8}.$$

4. $$\lim_{x\to 0^+}\left(\frac{\sqrt{1+\sin x}-1}{\sqrt{x}}\right)\left(\frac{\sqrt{1+\sin x}+1}{\sqrt{1+\sin x}+1}\right)$$

$$= \lim_{x\to 0^+} \frac{1+\sin x-1}{\sqrt{x}(\sqrt{1+\sin x}+1} = \lim_{x\to 0^+} \frac{\sin x}{\sqrt{x}(\sqrt{1+\sin x}+1)}$$

$$= \lim_{x\to 0^+} \frac{\sqrt{\sin x}\sqrt{\sin x}}{\sqrt{x}(\sqrt{1+\sin x}+1)} = \lim_{x\to 0^+}\left(\sqrt{\frac{\sin x}{x}}\right)\left(\frac{\sqrt{\sin x}}{\sqrt{1+\sin x}+1}\right)$$

$$= \sqrt{\lim_{x\to 0^+}\frac{\sin x}{x}} \;\; \lim_{x\to 0} \frac{\sqrt{\sin x}}{\sqrt{1+\sin x}+1}$$

$$= \sqrt{1}\;\frac{\sqrt{\sin 0}}{\sqrt{1+\sin 0}+1} = 1\cdot\frac{0}{2} = 0.$$

4 on practice sheet

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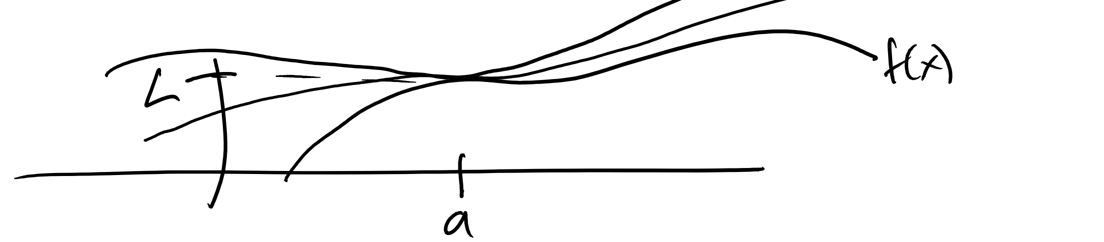
$$\lim_{x\to 0^+} x\sin(x^3-\ln x) = 0 \quad ?$$

Squeeze theorem: if $f(x)\le g(x)\le h(x)$ and $\lim_{x\to a} f(x) = L = \lim_{x\to a} h(x)$ then $\lim_{x\to a} g(x) = L$ also.



want: $-x \le x\sin(x^3-\ln x) \le x$

claim: $-x \le x\sin(x^3-\ln x) \le x$ since

?

$-1 \le \sin(x^3-\ln x) \le 1$

mult. by x

(ok since $x>0$ $\lim_{x\to 0^+}$

$$\lim_{x\to 0^+} -x = 0 = \lim_{x\to 0^+} x \implies \lim_{x\to 0^+} x\sin(x^3-\ln x) = 0$$

$$f(x) = \frac{1+\sqrt{x}}{x-1}$$

$$f'(x) = \lim_{h\to 0} \frac{\left(\frac{1+\sqrt{x+h}}{x+h-1}\right) - \left(\frac{1+\sqrt{x}}{x-1}\right)}{h}$$

strategy 1, common denominator simplify

strategy 2 break fractions up.

strategy 3 substitution

$$\lim_{h\to 0} \frac{\left(\frac{1}{x+h-1} + \frac{\sqrt{x+h}}{x+h-1} - \frac{1}{x-1} - \frac{\sqrt{x}}{x-1}\right)}{h}$$

$$\lim_{h\to 0} \frac{\left(\frac{1}{x+h-1} - \frac{1}{x-1}\right)}{h} + \lim_{h\to 0} \frac{\left(\frac{\sqrt{x+h}}{x+h-1} - \frac{\sqrt{x}}{x-1}\right)}{h}$$

$$\lim_{h\to 0} \frac{\left(\frac{x-1}{(x-1)(x+h-1)} - \frac{(x+h-1)}{(x-1)(x+h-1)}\right)}{h}$$

$$+\lim_{h\to 0} \frac{(x-1)\sqrt{x+h} - (x+h-1)\sqrt{x}}{h(x-1)(x+h-1)}$$

$$= \lim_{h\to 0} \frac{x-1-x-h+1}{h(x-1)(x+h-1)} +$$

$$\lim_{h\to 0}\left(\frac{(x-1)\sqrt{x+h} - (x+h-1)\sqrt{x}}{h(x-1)(x+h-1)}\right)\left(\frac{(x-1)\sqrt{x+h} + (x+h-1)\sqrt{x}}{(x-1)\sqrt{x+h} + (x+h-1)\sqrt{x}}\right)$$

$$\ldots^2 (\ldots) - (x+h-1)^2 x$$

$$\lim_{h\to 0}\left(\frac{\quad}{h(x-1)(x+h-1)}\right.$$

$$= \lim_{h\to 0}\frac{-h}{h(x-1)(x+h-1)} + \lim_{h\to 0}\frac{(x-1)^2(x+h) - (x+h-1)^2 x}{h\underbrace{(x-1)(x+h-1)\left((x-1)\sqrt{x+h} + (x+h-1)\sqrt{x}\right)}_{\text{STUFF}}}$$

$$= \frac{-1}{(x-1)^2} + \lim_{h\to 0}\frac{(x-1)^2 x + (x-1)^2 h - \left((x-1)+h\right)^2 x}{h\cdot \text{STUFF}}$$

$$= -\frac{1}{(x-1)^2} + \lim_{h\to 0}\frac{\cancel{(x-1)^2 x} + (x-1)^2 h - \cancel{x(x-1)^2} - 2x(x-1)h - h^2 x}{h\cdot\text{STUFF}}$$

$$= -\frac{1}{(x-1)^2} + \lim_{h\to 0}\frac{(x-1)^2\cancel{h} - 2x(x-1)\cancel{h} - h^{\cancel{2}} x}{\cancel{h}\cdot\text{STUFF}}$$

$$= -\frac{1}{(x-1)^2} + \lim_{h\to 0}\frac{(x-1)^2 - 2x(x-1) - h x}{(x-1)(x+h-1)\left((x-1)\sqrt{x+h} + (x+h-1)\sqrt{x}\right)}$$

$$= -\frac{1}{(x-1)^2} + \frac{(x-1)^2 - 2x(x-1)}{2(x-1)^2\left((x-1)\sqrt{x}\right)}$$

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6. $x = y + e^y$ if $y = 1 \Rightarrow x = 1 + e$

find $\frac{dy}{dx}$ when $x = 1 + e$.

^{y}

$$x = y + e^y$$

$$\frac{d}{dx} x = \frac{d}{dx}(y + e^y)$$

$$1 = \frac{dy}{dx} + e^y \cdot \frac{dy}{dx}$$

$$1 = \frac{dy}{dx}(1 + e^y)$$

$$\boxed{\frac{dy}{dx} = \frac{1}{1+e^y}} = \frac{1}{1+e^1} = \frac{1}{1+e}$$

want to find $\frac{dy}{dx}$ when $x = 1+e$

$$\Downarrow$$

$$y = 1$$