Lecture 14: Applications of Chain rule, continued

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Chapter 6: Chain Rule try to read this this week (by Thurs)

Derivative of logarithms

Remember: $\log_b x = y$ means $b^y = x$

Properties: $\log_b zw = \log_b z + \log_b w$

$$\log_b z^w = w \log_b z$$

$$\ln x = \log_e x$$

$\ln x = y$ means $\boxed{e^y = x}$ $y = \ln x$

$$e^{\ln x} = x$$

Since $e^{\ln x} = x$

$$\frac{d}{dx} e^{\ln x} = \frac{d}{dx} x$$

$$e^{\ln x} \boxed{\frac{d}{dx} \ln x} = 1$$

$$\frac{d}{dx} \ln x = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

$$\boxed{\frac{d}{dx} \ln x = \frac{1}{x}}$$

examples

$$\frac{d}{dx}\ln\left(e^{\sin x}\right) = \frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\ln x^2 = \frac{d}{dx}2\ln x = \frac{2}{x}$$

$$\frac{d}{dx}\ln(x^2+1) = \frac{1}{(x^2+1)}\cdot\frac{d}{dx}(x^2+1)$$

$$= \frac{2x}{x^2+1}$$

$$\frac{d}{dx}\ln\Big((x^2+1)(x^2+2)(x^2+3)(x^2+4)\Big) = \frac{1}{(x^2+1)(x^2+2)(x^2+3)(x^2+4)}\cdot\frac{d}{dx}\Big(\underbrace{(x^2+1)(x^2+2)\cdots}_{\text{product rule.}}\Big)$$

$$\parallel$$

$$\frac{d}{dx}\Big(\ln(x^2+1)+\ln(x^2+2)+\ln(x^2+3)+\ln(x^2+4)\Big)$$

$$= \frac{1}{x^2+1}(2x) + \frac{1}{x^2+2}(2x) + \frac{1}{x^2+3}(2x) + \frac{1}{x^2+4}(2x)$$

Change of base formulas

$$\frac{d}{dx}e^x = e^x \qquad \frac{d}{dx}\ln x$$

$$\frac{d}{dx}2^x \qquad \frac{d}{dx}\log_{10}x$$

Change of base for exponentials

given 2 "bases" a, b,

convert from one to the other.

$b^x = a^?$ take $\log_a$ of both sides

$\log_a b^x = \log_a a^? = ?$

$\overset{\shortparallel}{x \log_a b}$

$\Longrightarrow ? = x\log_a b$

$$\boxed{b^x = a^{x\log_a b}}$$

In particular: if $a = e$ $\Rightarrow$ $\boxed{b^x = e^{x\ln b}}$

ex: $\frac{d}{dx} 2^x = \frac{d}{dx} e^{x\ln 2} = e^{x\ln 2} \cdot \frac{d}{dx}(x\ln 2)$

$$= \underbrace{e^{x\ln 2}}_{\overset{\shortparallel}{2^x}} \ln 2 = 2^x \ln 2$$

Conversion Practice:

$2^x = e^{x\ln 2}$ $\qquad$ $3^{x^2} = e^{(x^2)\ln 3}$

$4^{\sin x} = e^{(\sin x)\ln 4}$ $\qquad$ $x^x = e^{x\ln x}$

$\frac{d}{dx} x^x = \frac{d}{dx} e^{x\ln x}$ $\qquad$ $\frac{d}{dx} x^3 = 3x^2$ $\qquad$ $\frac{d}{dx}(\;)^{\text{const}}$

$\frac{d}{dx}$ $\frac{d}{dx}$ $\frac{d}{dx}$ $\frac{d}{dx}$

$= e^{x\ln x} \cdot \frac{d}{dx}(x\ln x)$ $\qquad \frac{d}{dx}e^x \qquad \frac{d}{dx}e^{(\text{fcn of } x)}$

$= \underbrace{e^{x\ln x}}_{=x^x}\left(x\cdot\frac{1}{x}+1\cdot\ln x\right)$

$= x^x(1+\ln x)$

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log conversion

$\log_a \leftrightarrow \log_b \qquad x = b^{\log_b x}$

$\log_a x = \log_a b^{\log_b x} = (\log_b x)(\log_a b)$

$\log_a x = (\log_b x)(\log_a b)$

$$\boxed{\log_b x = \frac{\log_a x}{\log_a b}}$$

ex: $\log_b x = \frac{\ln x}{\ln b} \qquad (a=e)$

$\log_2 x = \frac{\ln x}{\ln 2}$ $\qquad \frac{d}{dx}\log_3 x = \frac{d}{dx}\frac{\ln x}{\ln 3}$

$= \frac{1}{\ln 3}\cdot\frac{1}{x}$

Concept of logarithmic differentiation

$$\frac{d}{dx}\underbrace{(x^2+1)(x^2+2)(x^2+3)(x^2+4)}_{y} = \text{what we want}$$

$$\frac{d}{dx}\ln y = \frac{1}{y}\cdot\boxed{\frac{d}{dx}y} = \frac{1}{(x^2+1)(x^2+2)(x^2+3)(x^2+4)}\cdot\frac{dy}{dx}$$

$$\| $$

$$\frac{1}{x^2+1}(2x) + \frac{1}{x^2+2}(2x) + \frac{1}{x^2+3}(2x) + \frac{1}{x^2+4}(2x)$$

$$\frac{dy}{dx} = \left((x^2+1)(x^2+2)(x^2+3)(x^2+4)\right)\cdot\left(\frac{1}{x^2+1}(2x) + \frac{1}{x^2+2}(2x) + \frac{1}{x^2+3}(2x) + \frac{1}{x^2+4}(2x)\right)$$

Principle: If we want to find $\frac{d}{dx}f(x)$, and we see that $\frac{d}{dx}\ln f(x)$ would be easier

Notice: $\frac{d}{dx}\ln f(x) = \frac{1}{f(x)}f'(x)$ (chain rule)

$$\| $$

(something relatively easy)

solve for $f'(x)$, get $\boxed{f'(x) = f(x)\cdot(\text{something relatively easy})}$

$(x+1)^{3/2}e^x$ $\quad$ $(x+1)^{3/2}e^x$

$$\frac{d}{dx}\underbrace{\frac{(x+1)^{3/2}e^{x}}{(x-1)^{5}\sqrt{x^{2}+3}}}_{f(x)}$$

$$\ln\frac{(x+1)^{7/2}e^{\wedge}}{(x-1)^{5}\sqrt{x^{2}+3}}$$

$$\shortparallel$$

$$\ln\left((x+1)^{3/2}e^{x}\right)-\ln(x-1)^{5}\sqrt{x^{2}+3}$$

$$=\ln(x+1)^{3/2}+\ln e^{x}-\ln(x-1)^{5}-\ln\sqrt{x^{2}+3}$$

$$=\ln(x+1)^{3/2}+\ln e^{x}-\ln(x-1)^{5}-\ln\sqrt{x^{2}+3}$$

$$=\frac{3}{2}\ln(x+1)+x-5\ln(x-1)-\frac{1}{2}\ln(x^{2}+3)$$

$$\frac{d}{dx}\ln\left(\frac{(x+1)^{3/2}e^{x}}{(x-1)^{5}\sqrt{x^{2}+3}}\right)=\frac{3}{2}\,\frac{1}{x+1}+1-5\,\frac{1}{x-1}-\frac{1}{2}\,\frac{2x}{x^{2}+3}$$

$$\shortparallel$$

$$\frac{1}{\left(\quad f(x)\quad\right)}\cdot\frac{d}{dx}\left(f(x)\right)$$

$$f'(x)=\left(\frac{(x+1)^{3/2}e^{x}}{(x-1)^{5}\sqrt{x^{2}+3}}\right)\left(\frac{3}{2}\,\frac{1}{x+1}+1-5\,\frac{1}{x-1}-\frac{1}{2}\,\frac{2x}{x^{2}+3}\right)$$