Lecture 16: applications of derivatives

Thursday, February 23, 2017 12:34 PM

$V = \pi R^2 H$

$R = 10$ cm
$H = 20$ cm

$V_{init} = \pi\, 10^2 \cdot 20 \approx 3 \cdot 100 \cdot 20$

$\approx 6{,}000$ cm^3

$\frac{dV}{dH} = \pi R^2 \approx 300$

(R const)

$H \to 21$ cm

$V_{fin} = \pi \cdot 10^2 \cdot 21 \approx 3 \cdot 100 \cdot 21$

increase by $1 \cdot (3 \cdot 100)$

$+1$ cm $H \rightsquigarrow +300$ vol. cm^3

H const

$\frac{dV}{dR} = \pi 2 R H \approx$

$3 \cdot 20 \cdot H$

$\approx 60 H \approx 600$

$H = 10$

$H = 20$
$R \to 11$

$\pi \cdot 11^2 \cdot 20 \approx 3 \cdot 121 \cdot 20$

$\approx 3 \cdot 120 \cdot 20$

≈ 720 more

$V = \pi R^2 H$

$R \to R + h$

$\pi (R+h)^2 H = \pi (R^2 + 2Rh + h^2) H$

$= \pi R^2 H + 2\pi R h H$

$+ \cancel{\pi h^2 H}$

ignore (very small)

$R \longrightarrow R+h$

$V \overset{\approx}{\rightsquigarrow} V + \underbrace{2\pi R H}_{\text{slope}} \cdot h$

?

slope

$V = \pi R^2 H$

$\frac{dR}{dV}$ (H const) $\quad \frac{dV}{dR}$ (H const) $\quad \frac{dH}{dR}$ (V const) ... $\quad \frac{dH}{dV} \quad \frac{dV}{dH}$ ...

$R = f(t) \qquad H = g(t)$

$t \to t+h \qquad R \to f(t+h) \approx f(t) + f'(t)h$

Basic idea: $\boxed{f(t+h) \approx f(t) + f'(t)h}$ "Linear approx"

why: $f'(t) = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$

$f'(t) \approx \frac{f(t+h) - f(t)}{h}$ if h is small

$hf'(t) \approx f(t+h) - f(t)$

$V = \pi R^2 H \qquad R = f(t) \qquad H = g(t)$

$\frac{dV}{dt} = \pi\left(2R\frac{dR}{dt}H + R^2\frac{dH}{dt}\right)$

$$\pi\left(\frac{d}{dt}(R^2)\,H + R^2\frac{d}{dt}H\right) \qquad R + \frac{dK}{dt}h$$

$$t \to t+h \qquad R \longrightarrow f(t+h) \approx f(t) + f'(t)h$$

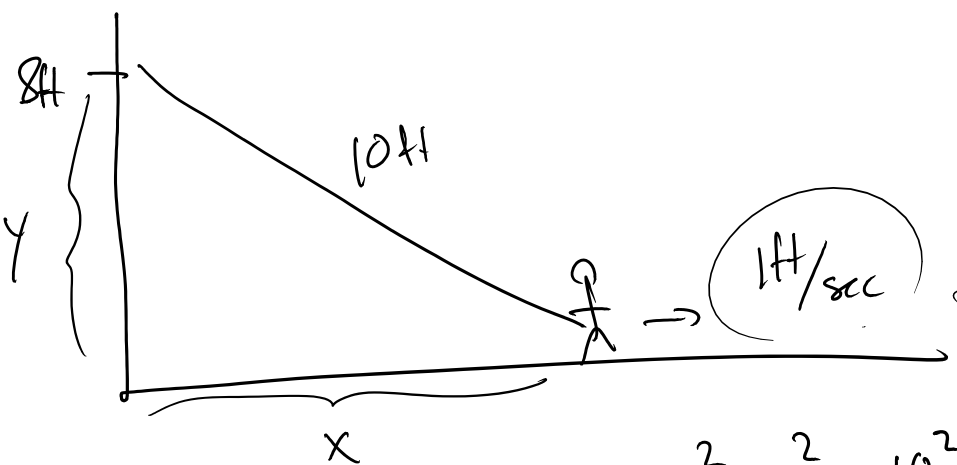
$$H \longrightarrow g(t+h) \approx g(t) + g'(t)h = H + \frac{dH}{dt}h$$

$$V \to \pi R^2 H = \pi\left(R + \frac{dR}{dt}h\right)^2\left(H + \frac{dH}{dt}h\right)$$

$$= \pi\left(R^2 + 2R\frac{dR}{dt}h + \underset{h^2}{\text{small}}\right)\left(H + \frac{dH}{dt}h\right)$$

$$= \pi R^2 H + \pi R^2\frac{dH}{dt}h + 2R\frac{dR}{dt}Hh + \cancel{h^2\text{ stuff}}$$

$$\approx V + \pi\left(R^2\frac{dH}{dt} + 2R\frac{dR}{dt}H\right)h$$



when ladder is 8ft up, how fast is it fally?

$$\frac{dx}{dt} = 1$$

$$x^2 + y^2 = 10^2$$

$\frac{d}{dt}$ both sids

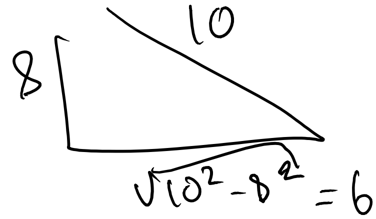
$$\frac{dy}{dt} = ?$$

$$\frac{d}{dt}(x^2) + \frac{d}{dt}(y^2) = \frac{d}{dt}10^2 = 0$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0 \qquad \frac{dy}{dt}=-\frac{2x}{2y}\frac{dx}{dt}$$

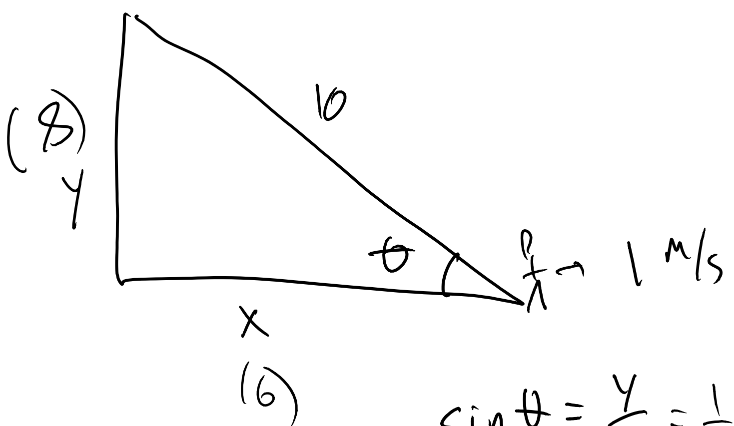
$$=-\frac{x}{y}\frac{dx}{dt}=-\frac{x}{y}$$

when $y=8$,



when $y=8$

$$\frac{dy}{dt}=-\frac{x}{8}=-\frac{6}{8}=-\frac{3}{4}$$



how fast is θ changing?

$$\frac{d\theta}{dt}$$

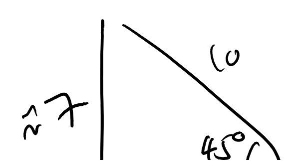
$$\sin\theta=\frac{y}{10}=\frac{1}{10}y$$

$$\cos\theta\frac{d\theta}{dt}=\frac{1}{10}\frac{dy}{dt}=\frac{1}{10}\left(-\frac{3}{4}\right)$$

$$\left(\frac{6}{10}\right)\frac{d\theta}{dt}=\frac{1}{10}\left(-\frac{3}{4}\right) \qquad \frac{d\theta}{dt}=-\frac{1}{8}$$

in 1 sec, ladder has fallen $\approx$ 9 in $\approx$ 1 ft   now $\approx$ 7 ft
ladder has moved 1 ft from wall   now $\approx$ 7 ft

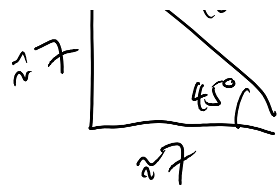
$\angle$ of ladder after 1 sec?



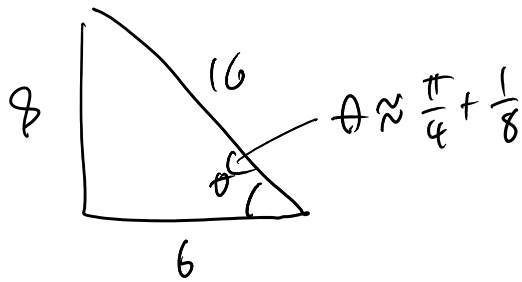
$\approx \frac{\pi}{4}$ after 1 sec?

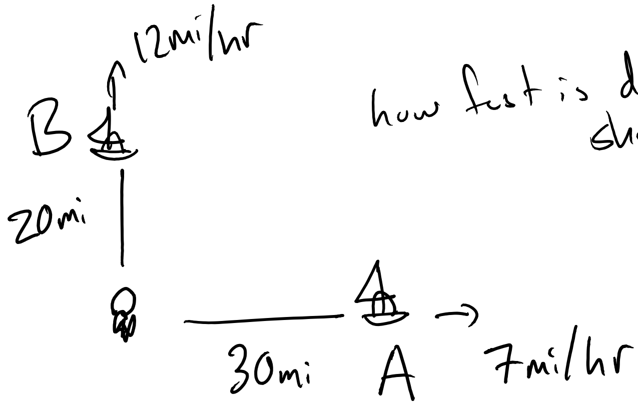
∡ of ladder

~~θ ≈~~ $\frac{\pi}{4}$ after 1 sec?

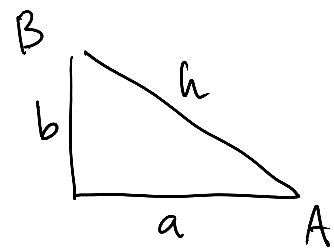


initial ∡ $\approx \frac{\pi}{4} + \frac{1}{8}$





how fast is distance between ships changing?



Assign variable names to

- things you want to find rates of change of
- " " know the " "

$\frac{dh}{dt}$ want

know $\frac{da}{dt} = 7$ $\frac{db}{dt} = 12$

Find equ to relate them

$$a^2 + b^2 = h^2$$

when $a = 30$ & $b = 20$

take $\frac{d}{dt}$'s. $2a\frac{da}{dt} + 2b\frac{db}{dt} = 2h\frac{dh}{dt}$

solve $\frac{dh}{dt} = \dfrac{a\frac{da}{dt} + b\frac{db}{dt}}{h} = \dfrac{30(7) + 20(12)}{\sqrt{30^2 + 20^2}}$