Related Rates

Common theme: - have some quantities changing (with time)
- these quantities are related (via geometry)
- know rate of change of one (or more)
- need to find rate of change of another.

Strategy

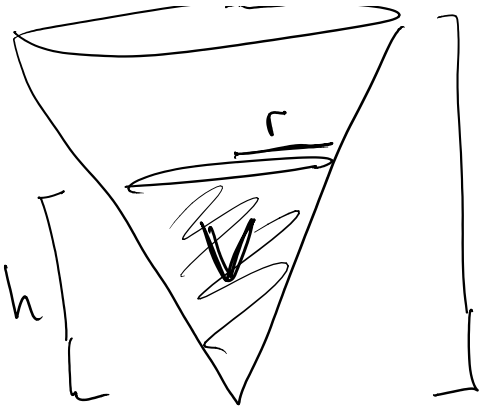
1. Read, read, read.
2. Draw a picture / diagram
2½. Picture needs to show variables of interest
   - thing you want rate of change of
   - things you know rates of change of.
3. Find an equation(s) relating the variables
   (don't need to solve for any particular variable here)
4. Implicit differentiation, solve for rate of change of interest
5. Plug in values

An inverted circular cone is being filled with frozen yogurt at a constant rate of $30\ cm^3/sec$. If the radius at the top of the cone is 30 cm and the height is 100 cm, find the rate at which the height of the frozen yogurt in the cone is increasing when the height is:

- 20 cm?
- 70 cm?

$V = \frac{1}{3}\pi r^2 h$

want ↓



$V = \frac{1}{3}\pi r^2 h$

want ↓

$\frac{dV}{dt} = \frac{1}{3}\pi\left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right)$

know $30\ cm^3/sec$

$\frac{dV}{dt} = \frac{1}{3}\pi 2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\frac{1}{3}\pi$

to solve:
- distribute
- separate
- factor
- divide

$\frac{dV}{dt} - \frac{1}{3}\pi 2r\frac{dr}{dt}h = \frac{r^2}{3}\pi\frac{dh}{dt}$

did this too soon

plug in 20 or 70

$$\frac{\left(\frac{dV}{dt} - \frac{2\pi}{3}r\frac{dr}{dt}h\right)}{\left(\frac{\pi r^2}{3}\right)} = \frac{dh}{dt}$$

30

r? $\frac{dr}{dt}$?

what's going on:
in a nutshell, too many variables, too few equations.

had variables r, h, V, knew about V (rate-style)

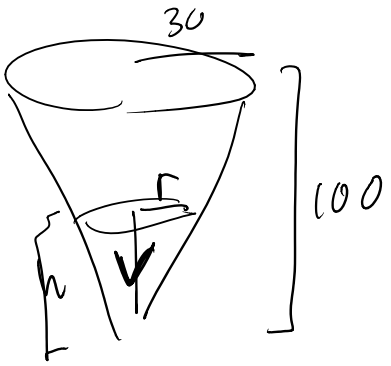
trying to solve for h

only had one eqn.

Need another equation

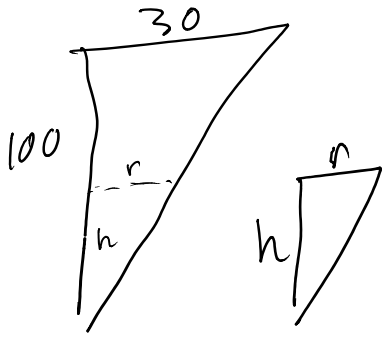
30

$V = \frac{1}{2}\pi r^2 h$



$V = \frac{1}{3}\pi r^2 h$

Similar $\triangle$'s!



$\frac{30}{100} = \frac{r}{h}$

$30h = 100r$ (can eliminate r's)

$\frac{d}{dt} \rightsquigarrow 30\frac{dh}{dt} = 100\frac{dr}{dt}$ (can eliminate $\frac{dr}{dt} \rightarrow \frac{dh}{dt}$)

Strategy 1: eliminate at beginning!

$V = \frac{1}{3}\pi r^2 h$ $\quad 30h = 100r$ $\quad r = \frac{3}{10}h$

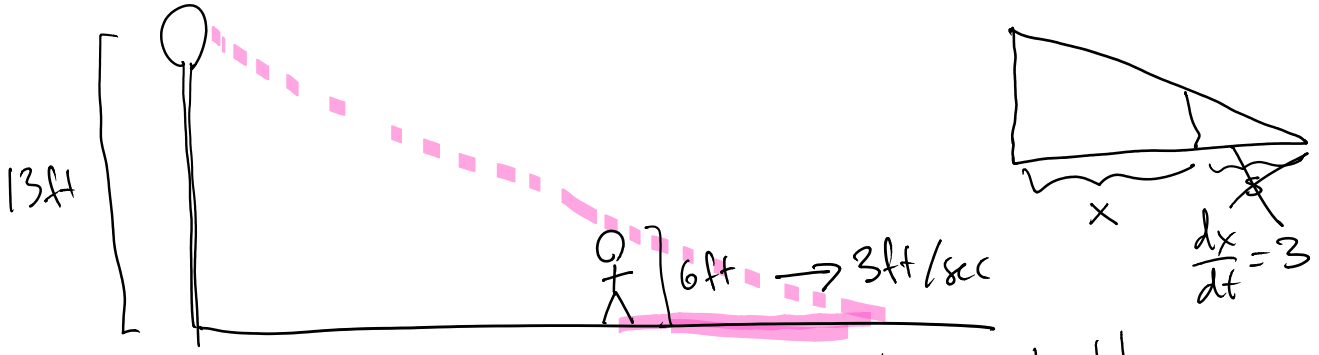
$= \frac{1}{3}\pi\left(\frac{3}{10}h\right)^2 h = \frac{1}{3}\pi\frac{9}{100}h^3$

$V = \frac{3\pi}{100}h^3$ $\xrightarrow{\frac{d}{dt}}$

$\frac{dV}{dt} = \frac{3\pi}{100}3h^2\frac{dh}{dt}$

$\rightarrow \frac{1}{\left(\frac{3\pi}{100}3h^2\right)}\frac{dV}{dt} = \frac{dh}{dt}$

20 or 70 $\quad$ 30



how fast is tip of shadow cast by street lamp moving when person is 10 ft from lamp-post?