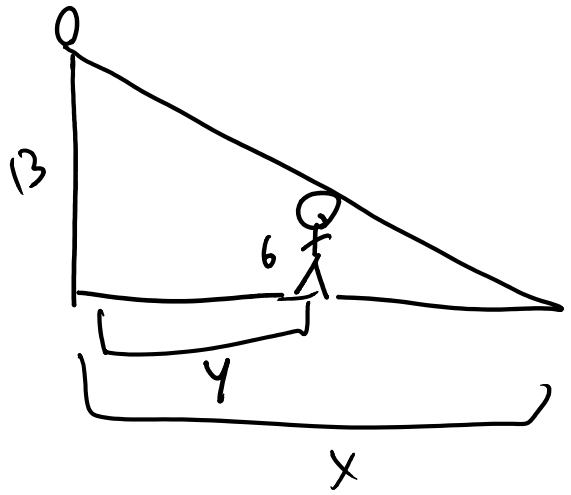
Lecture 18: Related Rates, part 2

Tuesday, February 28, 2017 12:39 PM

Next exam topics (roughly)

- Log. diff
- All derivatives (inverse trig fcns, log, exp (w/ different bases))
- Some approximations ($f(x+h) \approx f(x) + f'(x)h$)
- Lots of related rates

A person is walking at night from a 13 ft tall lamp post, at a speed of 3 ft/sec. If they are 6 ft tall, how quickly is the top of their shadow moving away from the lamppost when the person is 10 ft away from from lamp post?



$$6/x-y = 13/x$$

$$6x = 13(x-y)$$

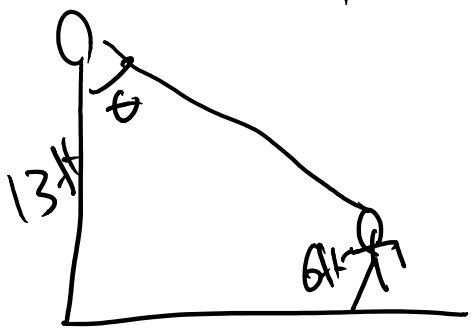
$$6x = 13x - 13y$$

$$13y = 7x$$

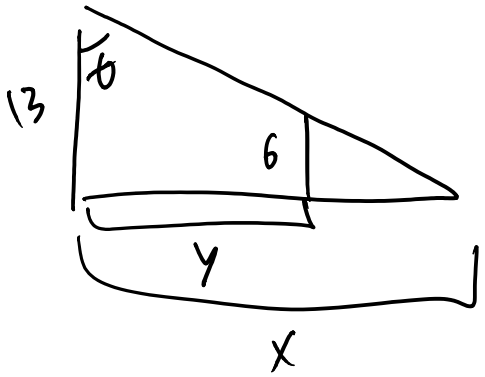
$$13\frac{dy}{dt} = 7\frac{dx}{dt}$$

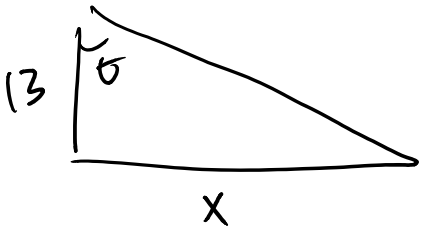
↑

In same problem, how fast is the angle changing between streetlamp & person's head?
(when they are 10 ft away)



→ 3 ft/sec

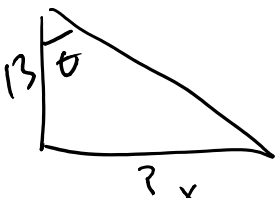


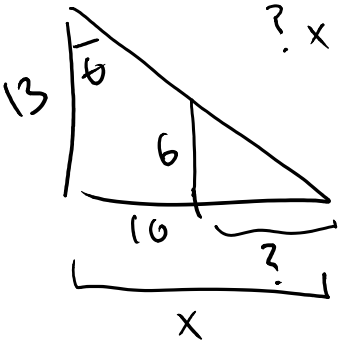


$$\tan\theta = \frac{x}{13}$$

$$\frac{d}{dt}\tan\theta = \frac{d}{dt}\frac{x}{13}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{13}\frac{dx}{dt}$$ ← found in last problem

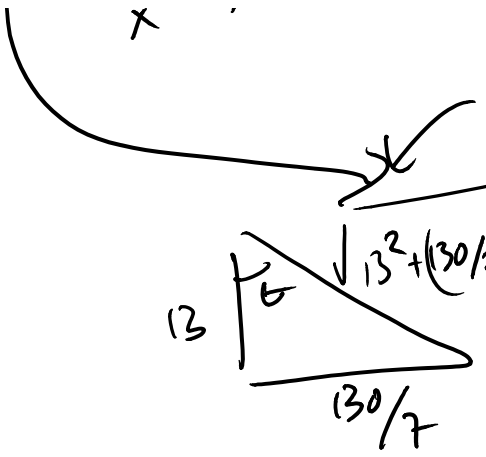




$$\frac{13}{x} = \frac{6}{x-10}$$

$$13(x-10) = 6x$$

$$13x - 130 = 6x$$



$13x - 130 = 6x$

$7x = 130$

$x = \frac{130}{7}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{13^2+(130/7)^2}}{13}$

$\sec^2\theta = \frac{13^2+(130/7)^2}{13^2}$

$$\left(\frac{13^2+(130/7)^2}{13^2}\right)\frac{d\theta}{dt} = \frac{1}{13}\frac{dx}{dt}$$

$dx/dt = 39/7$

$$\frac{d\theta}{dt} = \left(\frac{1}{13}(39/7)\right) \Big/ \left(\frac{13^2+(130/7)^2}{13^2}\right)$$

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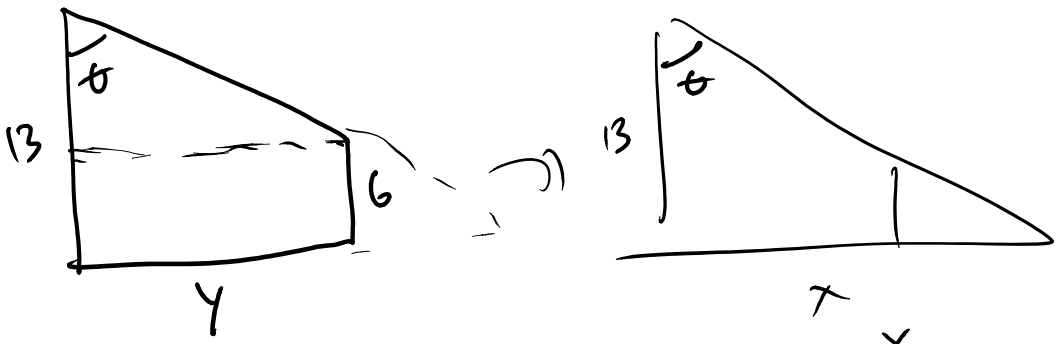
or:



$\tan\theta = \frac{x}{13}$

$\tan\theta = \frac{13y/7}{13}$

$\frac{x}{13} = \frac{x-y}{6}$

$6x = 13x - 13y$

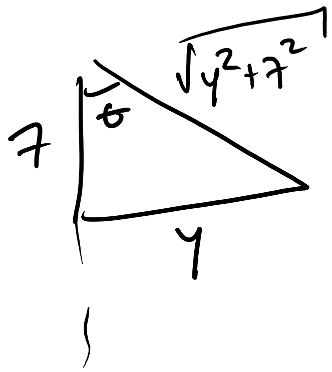
$-7x$

$\tan\theta = \frac{(\ldots/7)}{13}$ 13 6 $6x = 13x \ldots$

$13y = 7x$

$\tan\theta = y/7$

$\frac{13y}{7} = x$



$\tan\theta = \frac{y}{7}$ implicit

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{7}\frac{dy}{dt}$

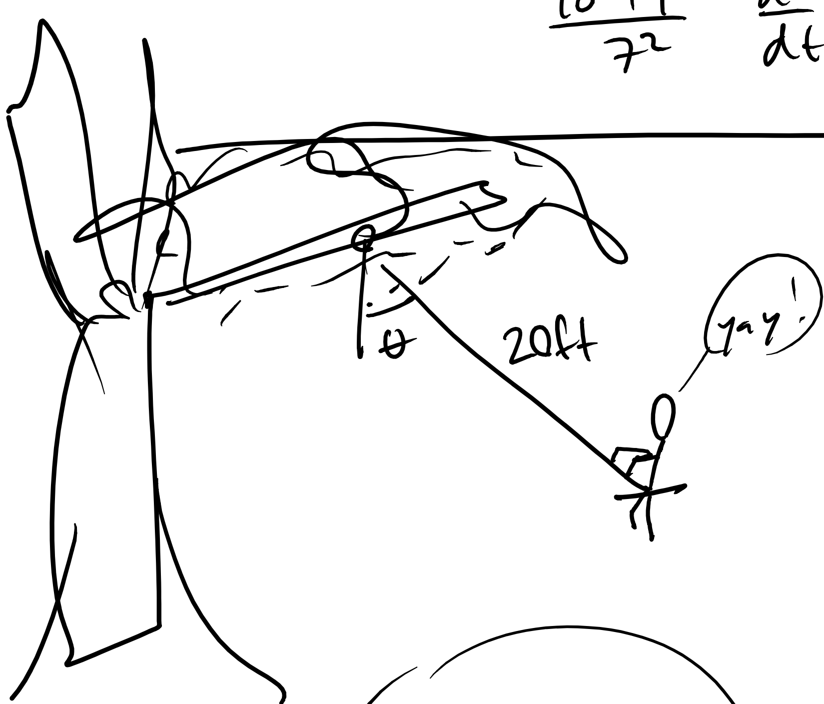
3 plug in

$\sec\theta = \frac{1}{\cos\theta}$

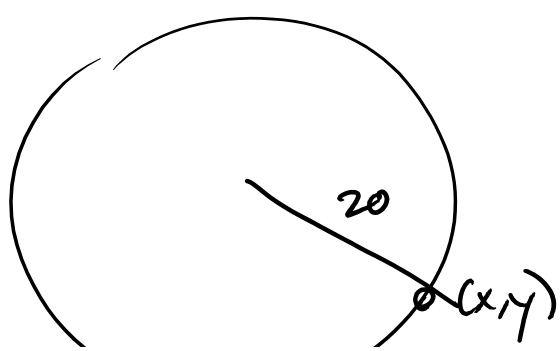
$= \frac{\sqrt{10^2+7^2}}{7}$

$\sec^2\theta = \frac{10^2+7^2}{7^2}$

$\frac{10^2+7^2}{7^2}\frac{d\theta}{dt} = \frac{1}{7}3$

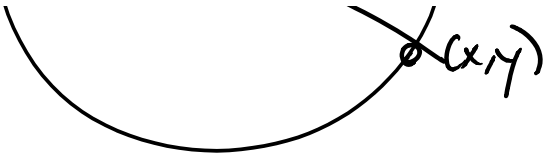


If person mov to left at 10 ft/sec how fast are they mov down when θ is 30° ?



$x^2+y^2=20^2$

want $\frac{dy}{dt}$, know $\frac{dx}{dt}$.



want $\frac{dy}{dt}$, know $\frac{dx}{dt}$.

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

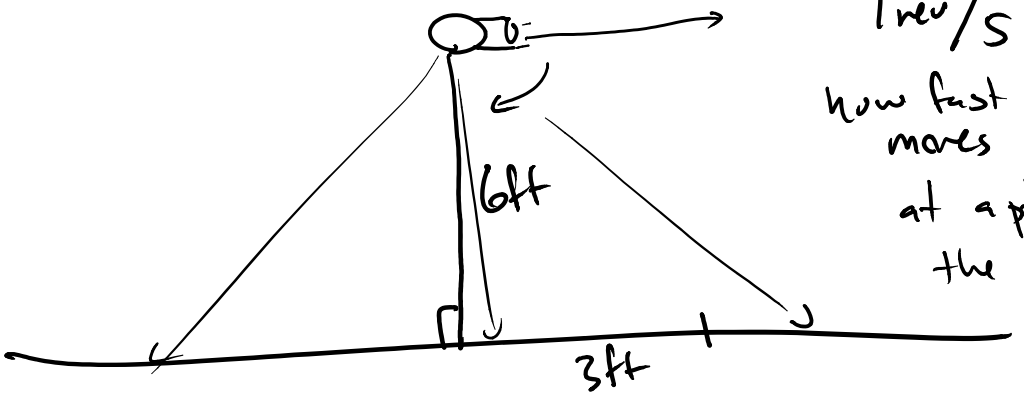
$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

-10

slick way: $\frac{x}{y} = \tan\theta$

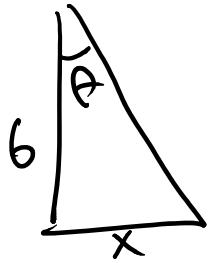
$= \tan 30^\circ = \sqrt{3}/3$

$$= \frac{\sqrt{3}}{3} \cdot 10$$



1 rev/5 sec

how fast the laser moves across wall at a pt 3ft from the perpendicular



want $\frac{dx}{dt}$, have $\frac{d\theta}{dt} = 2\pi/5\,\text{sec}$