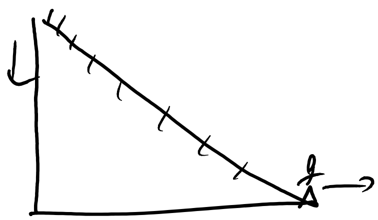


Problems

compare rate of x & y

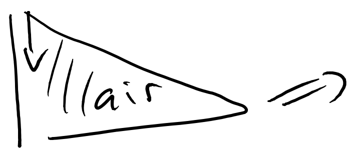
" " x & θ

x & Area

Also could ask:

when is area as large as possible?

how fast is air escaping



when is air escaping fastest?

Could have more interesting descriptions of motion:

ex: $\frac{dy}{dt} = y - 1$ if $y = 1$ (vertical)  not moving.

$y < 1$  $\frac{dy}{dt} = (\text{less than } 1) - 1 = \text{neg}$

falling.

Real world "related rates"

Population dynamics

More rabbits = faster growing rabbit population.

$R = \#$ rabbits $\qquad \frac{dR}{dt} =$ rabbit growth rate.

R = # rabbits $\qquad \frac{dR}{dt}$ = rabbit growth rate.

example $\quad \frac{dR}{dt} = R \quad$ or $\quad \frac{dR}{dt} = 2R$

$\frac{dR}{dt} = R^2 \qquad \cancel{\frac{dR}{dt} = e^R}$?

0 rabbits
$\Rightarrow$ 0 growth!

F = # foxes

more rabbits = faster rabbit growth
more foxes = more rabbit mortality.

$$\frac{dR}{dt} = 2R - F^2$$ example.

more rabbits
"
faster fox growth

$$\frac{dF}{dt} = RF - F^2 = (R-F)F$$

$R - F$?

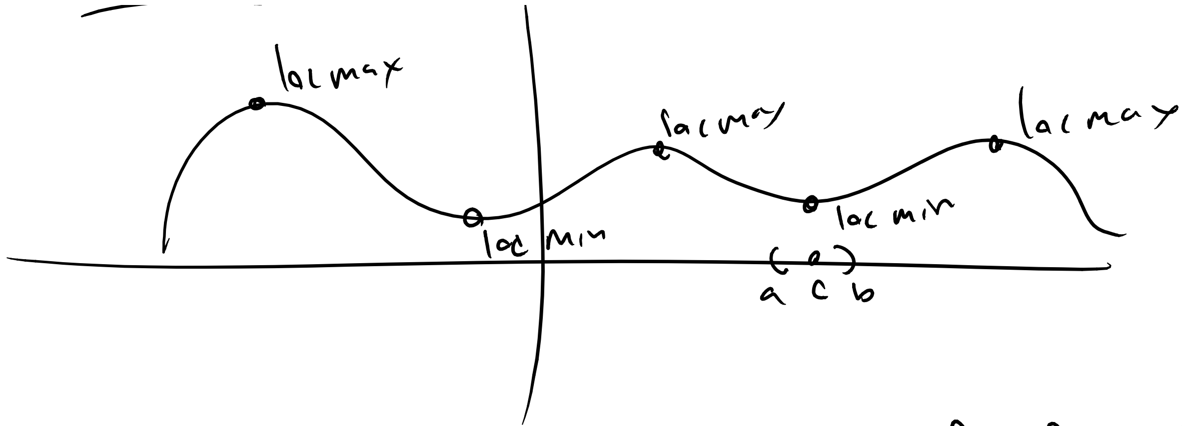
more foxes
" ?

what's max number of rabbits pop will have?
when

Q: How to optimize / find min & max values?

Basic idea: Notion of local minima & maxima

loc max

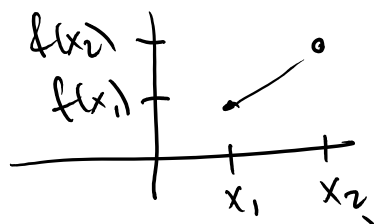


<u>Def</u> a number c is a local min for $f(x)$ if for some interval (a,b) containing c, we have $f(x) \geq f(c)$ for all x in (a,b).

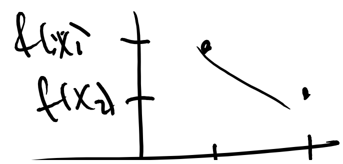
<u>Def</u> Similarly c a local max if - - — — - - — — — — — — - - $f(x) \leq f(c)$ all x in (a,b)

Non-extrema: (increasing / decreasing)

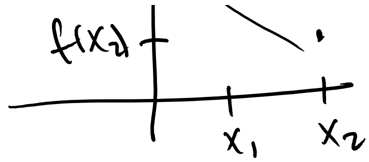
<u>Def</u> $f(x)$ is increasing on an interval (a,b) if for any x_1, x_2 in (a,b) with $x_1 < x_2$ we have $f(x_1) < f(x_2)$



<u>Def</u> $f(x)$ is decreasing on an interval (a,b) if for any x_1, x_2 in (a,b) with $x_1 < x_2$ we have $f(x_1) > f(x_2)$

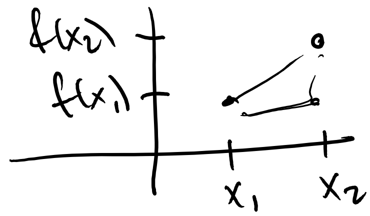


$f(x_1) > f(x_2)$



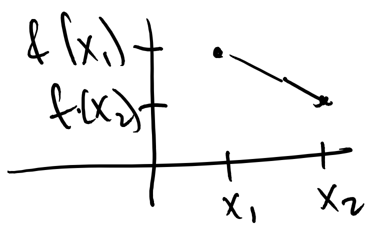
<u>Def</u> $f(x)$ is nondecreasing on an interval (a,b) if for any x_1, x_2 in (a,b) with $x_1 < x_2$ we have

$f(x_1) \leq f(x_2)$



<u>Def</u> $f(x)$ is nonincreasing on an interval (a,b) if for any x_1, x_2 in (a,b) with $x_1 < x_2$ we have

$f(x_1) \geq f(x_2)$



<u>Main Fact</u> (will explain fully later)

- If $f(x)$ is a function & $f'(c) > 0$ then $f(x)$ is increasing on some interval around c (and so, c is <u>not</u> a local min or max)

- . . — — — — $f'(x) < 0$ — — - decreasing

— — — — — — —

<u>Conclusion</u> is: Only possible local mins or max's occur

. . . neither $f'(x) > 0$ nor when $f'(x) < 0$

only when neither $f'(x)>0$ nor when $f'(x)<0$

Said backwards? local min/max can only occur at points where either

- $f'(x)=0$ or
- $f'(x)$ does not exist

So if $f'(c)=0$ or $f'(c)$ d.n.e we say that c is a critical point

(c in interior of domain of $f(x)$)

example: Find local extrema of $f(x)=x^2-5x-1$

Step 1: locate crit pts:

$f'(x)=2x-5$ when is $f'(x)=0$

" " $f'(x)$ not defined?

defined everywhere.

$f'(x)=0=2x-5 \Rightarrow \boxed{x=\frac{5}{2} \text{ crit pt}}$

when is $f'(x)>0$?

when is $f'(x)<0$?

$2x-5>0$

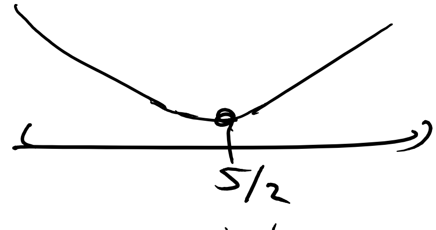
$2x>5$ incr

$x>5/2$

$2x-5<0$

decresg $\rightarrow$ $2x<5$

$x<5/2$



so must be a min!

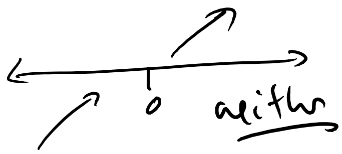
ex: $f(x) = x^3$

$f'(x) = 3x^2$

always pos if $x \neq 0$

always defined

$f'(x) = 0 = 3x^2 \Rightarrow \boxed{x=0}$ crit pt.



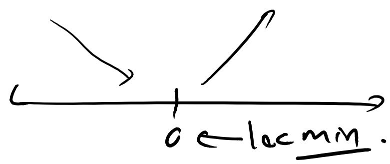
$f(x) = (\sqrt[3]{x})^2$

$f(x) = x^{2/3}$

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3}\frac{1}{\sqrt[3]{x}}$

$f'(x)$ not defined at $\boxed{x=0}$ crit pt

$f'(x) = 0$ never.



$f'(x) = \frac{2}{3}\frac{1}{\sqrt[3]{x}} > 0$ if $x > 0$

$f'(x) = \frac{2}{3}\frac{1}{\sqrt[3]{x}} < 0$ if $x < 0$

$f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -x^{-2} = -\frac{1}{x^2}$

when is $f'(x)$ not defined @ $x=0$. not a crit pt. (not in domain)