Lecture 2: Limits

Tuesday, January 10, 2017 2:00 PM

text: 1.1, 1.2, little 1.3

$$x = y \qquad x, y \neq 0$$
$$\downarrow 1$$
$$x^2 = xy$$
$$\downarrow 2$$
$$x^2 - y^2 = xy - y^2$$
$$\downarrow 3$$
$$(x+y)(x-y) = y(x-y)$$
$$\downarrow 4 \quad \longleftarrow \div 0 \text{!!}$$
$$x + y = y$$
$$\downarrow 5 \quad \downarrow x = y$$
$$2y = y$$
$$\downarrow 6$$
$$2 = 1 \quad ?$$

Goal: try to say things that make sense.

If we want to understand "rate of change at a point"

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$ ← how much is x actually changing? its not.

need a new concept to avoid $\div 0$!

Limits

We say that the limit of $f(x)$ as x approaches a is L to mean that: $f(x)$ gets close to L whenever x gets close to (but not equal to) a.

<u>ex:</u> $f(x) = x/2$ as x approaches 4, f(x) approaches 2

limit of $x/2$ as x approaches 4 is 2.

if $|x-4|<2 \Rightarrow -2 < x-4 < 2$

$$\Downarrow$$

$$-1 < \frac{x}{2} - 2 < 1$$

$$\Downarrow$$

$$|x/2 - 2| < 1$$

can we ensure $|f(x) - 2| < \frac{1}{4}$?

sure. $|x-4| < \frac{1}{2}$

$-\frac{1}{2} < x-4 < \frac{1}{2}$

$-\frac{1}{4} < \frac{x}{2} - 2 < \frac{1}{4} \nearrow |\frac{x}{2} - 2| < \frac{1}{4}$.

It doesn't matter what number you ask: we can always do it!

$$\downarrow$$

$$-2\varepsilon < x-4 < 2\varepsilon = \delta$$

$-\varepsilon < \frac{x}{2} - 2 < \varepsilon \Rightarrow |\frac{x}{2} - 2| < \varepsilon$.

how close can we make it? as close as we want.

Definition $\lim_{x \to a} f(x) = L$ (limit of $f(x)$ as x gets close to a is L)

means, for any $\varepsilon > 0$, there is a number $\delta > 0$ so that $|f(x) - L| < \varepsilon$ whenever $0 < |x-a| < \delta$

ex: $\lim_{x \to 4} \frac{x}{2} = 2$ showed this by noticing that for any $\varepsilon > 0$, if we let $\delta = 2\varepsilon$ then

if $0 < |x-4| < \delta = 2\varepsilon$ then $\left|\frac{x}{2} - 2\right| < \frac{\delta}{2} = \varepsilon$

Compute limits

Limit Facts

1. $\lim_{x \to a} C = C$ $\quad$ C constant

2. $\lim_{x \to a} x = a$

Limit Laws

Suppose $f(x), g(x)$ are functions and $\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$

then:

1. $\lim (f(x) + g(x)) = L + M$

then:

1. $\lim_{x\to a}\left(f(x)+g(x)\right) = L+M$

2. $\lim_{x\to a} f(x)g(x) = LM$

3. $\lim_{x\to a} Cf(x) = C\lim_{x\to a} f(x)$

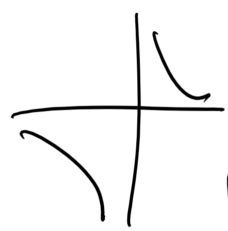
4. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{L}{M}$ if $M \neq 0$

Is it true that $\lim_{x\to a} f(x)+g(x) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$?

no.

4) $\lim_{x\to 0} \frac{x}{x} = 1$ (can't use rule 4)

$\lim_{x\to 0} \left(\frac{1}{x} + \left(-\frac{1}{x}\right)\right) = 0$



$\lim_{x\to 0} \frac{1}{x} = \text{d.n.e.}$

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ex: $\lim_{x\to 2} 3x^2-1 = \lim_{x\to 2} 3x^2 + (-1)$

(only if limits make sense)

$= \lim_{x\to 2} 3x^2 + \lim_{x\to 2}(-1)$

$= 3\lim_{x\to 2} x^2 + (-1)$

$= 3\left(\lim_{x\to 2} x\right)\left(\lim_{x\to 2} x\right) + (-1)$

$$= 3(2)(2) - 1 = 3(2)^2 - 1$$

<u>Practice</u>

$$\lim_{x \to 3} \frac{(x^2-9)\,2x}{x-3} = \lim_{x \to 3} \frac{(x+3)(x-3)\,2x}{x-3}$$

(limit)
$x \neq 3$
$x - 3 \neq 0$
whew!

$$\lim_{x \to 3} (x+3)\,2x$$

$$= \left[\lim_{x - 3} (x+3)\right]\left[\lim_{x \to 3} 2x\right] = \left[\lim_{x \to 3} x + \lim_{x \to 3} 3\right]\left[2 \lim_{x \to 3} x\right]$$

$$= (3+3)(2 \cdot 3) = 36$$