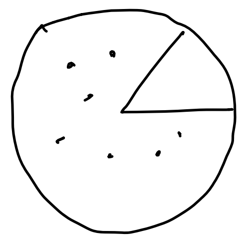
3/14/2017

3.14 PIE

π



Math 2250, Spring 2017, Practice Sheet for Exam 2

---

(1) Find the derivative of the function

$$f(x) = \frac{x^x(x^2-4)^5(x-1)^3}{e^x\sqrt{x+e^x}}$$

(2) Solve for $\frac{dy}{dx}$ given that

$$x^2 - 4y^2 = \sin(xy)$$

(3) Solve for $\frac{dx}{dt}$ given that

$$e^{xy} + \ln(x+y) = y$$

(4) Given that

$$x + 3y - \sin(y) = 17, \text{ and } \frac{dy}{dt} = 3$$

solve for $\frac{dx}{dt}$ when $y = 0$.

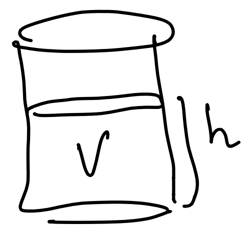
(5) Water is draining from a cylindrical tank with a radius of 100cm at a constant rate of 20cm^3/min. Find the rate of change of the height of the water in the tank when the water's height is 500cm.
Recall that the formula for the volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius and h is the height

(6) Water is in a conical tank with a height of 800cm and radius at the top of 300cm. If the water is draining out at a constant rate of 20cm^3/min, find the rate of change of the height of the water in the tank when the water's height is 100cm.
Recall that the formula for the volume of a circular cone is given by $V = \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height

(7) Water is draining from a cylindrical tank with a radius of 100cm at a rate inversely proportional to the amount of water in the tank given by the formula:

$$\frac{dV}{dt} = -1/10V \qquad -\frac{1}{10V} \qquad -\frac{1}{10}V$$

where V is the volume of the tank in cc's (cubic centimeters), and where the speed is given in cc/sec. Find the rate of change of the height of the water in the tank when the water's height is 500cm.



$$V = \pi r^2 h$$

$$V = \pi (100)^2 h$$

$$\frac{dV}{dt} = \pi (100)^2 \frac{dh}{dt}$$

know

$$-\frac{1}{10}V$$

$$\pi r^2 h$$

100 500

(8) Consider the function $f(x) = x(6 - 2x)^2$.
 (a) Find all the critical points of $f(x)$
 (b) Find the intervals on which $f(x)$ is increasing or decreasing
 (c) Find all local minima and maxima
 (d) Find all absolute minima and maxima

(9) Suppose that we have functions $f(x), g(x)$ such that

$$f(g(x)) - 3g(x) + 2f(x + f(x)) = 9x.$$

Solve for $g'(x)$.

$$\frac{dh}{dt} = \frac{1}{\pi(100)^2}\frac{dV}{dt}$$

$$\frac{dV}{dt} = -\frac{1}{10}(100)^2(500)\pi$$

1. (0 pts) When a circular plate of metal is heated in an oven, its radius increases at a rate of 0.03 cm/min. At what rate is the plate's area increasing when the radius is 40 cm?

Answer = ______________ cm^2/min

Answer(s) submitted:

-

(incorrect)

2. (1 pt) The length l of a rectangle is decrasing at a rate of 3 cm/sec while the width w is increasing at a rate of 4 cm/sec. When l = 11 cm and w = 9 cm, find the following rates of change:

The rate of change of the area:

Answer = ______________ cm^2/sec.

The rate of change of the perimeter:

Answer = ______________ cm/sec.

The rate of change of the diagonals:

Answer = ______________ cm/sec.

Answer(s) submitted:

-
-
-

(incorrect)

3. (1 pt) Two commercial airplanes are flying at an altitude of 40,000 ft along straight-line courses that intersect at right angles. Plane A is approaching the intersection point at a speed of 423 knots (nautical miles per hour; a nautical mile is 2000 yd or 6000 ft.) Plane B is approaching the intersection at 441 knots.

At what rate is the distance between the planes decreasing when Plane A is 4 nautical miles from the intersection point and Plane B is 4 nautical miles from the intersection point?

Answer = ______________ knots.

Answer(s) submitted:

-

(incorrect)

4. (1 pt) Sand falls from a conveyor belt at a rate of 30 m^3/min onto the top of a conical pile. The height of the pile is always $\frac{3}{4}$ of the base diameter. Answer the following.

a.) How fast is the height changing when the pile is 9 m high?

Answer = ______________ m/min.

b.) How fast is the radius changing when the pile is 9 m high?

Answer = ______________ m/min.

Answer(s) submitted:

-
-

(incorrect)

5. (1 pt) When air expands adiabatically (without gaining or losing heat), its pressure P and volume V are related by the equation $PV^{1.4} = C$ where C is a constant. Suppose that at a certain instant the volume is 500 cubic centimeters and the pressure is 97 kPa and is decreasing at a rate of 14 kPa/minute. At what rate in cubic centimeters per minute is the volume increasing at this instant?

Answer: ______________

Note: Pa stands for Pascal. One Pa is equivalent to one Newton$/\text{m}^2$. kPa is a kiloPascal or 1000 Pascals.

Answer(s) submitted:

-

(incorrect)

6. (1 pt) A spherical snowball is melting in such a way that its diameter is decreasing at rate of 0.3 cm/min. At what rate is the volume of the snowball decreasing when the diameter is 8 cm. (Note the answer is a positive number).

____

Answer(s) submitted:

-

(incorrect)

7. (1 pt) The length of a rectangle is increasing at a rate of 6cm/s and its width is increasing at a rate of 3cm/s. When the length is 30cm and the width is 20cm, how fast is the area of the rectangle increasing?

Answer (in cm^2/s): ______________

Answer(s) submitted:

-

(incorrect)

8. (1 pt) Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 2m/s, how fast is the area of the spill increasing when the radius is 15m?

Answer (in m^2/s): ______________

Answer(s) submitted:

-

(incorrect)

9. (1 pt) A spotlight on the ground is shining on a wall 24m away. If a woman 2m tall walks from the spotlight toward the building at a speed of 0.8m/s, how fast is the length of her shadow on the building decreasing when she is 8m from the building?

Answer (in meters per second): ______________

Answer(s) submitted:

-

(incorrect)

10. (1 pt) If $z^2 = x^2 + y^2$ with $z > 0$, $dx/dt = 4$, and $dy/dt = 5$, find dz/dt when $x = 8$ and $y = 15$.

Answer: $\dfrac{dz}{dt} =$ ______________

Answer(s) submitted:

-

(incorrect)

11. (1 pt)

A particle is moving along the curve $y = \sqrt{x}$. As the particle passes through the point $(4,2)$, its x-coordinate increases at a rate of $3\,cm/s$. How fast is the distance from the particle to the origin changing at this instant?

_________ cm/s

Answer(s) submitted:

-

(incorrect)

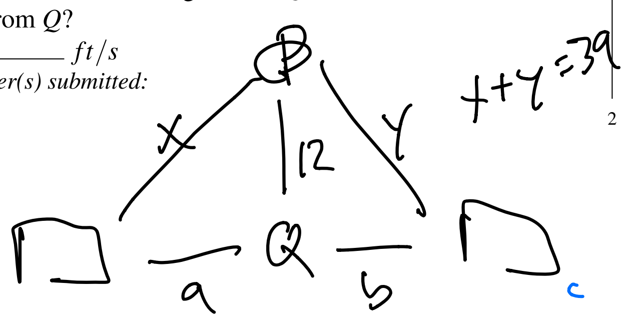
12. (0 pts)

Two carts, A and B, are connected by a rope $39\,ft$ long that passes over a pulley P. The point Q is on the floor $12\,ft$ directly beneath P and between the carts. Cart A is being pulled away from Q at a speed of $2\,ft/s$.

How fast is cart B moving toward Q at the instant when cart A is $5\,ft$ from Q?

_________ ft/s

Answer(s) submitted:



-

(incorrect)

13. (0 pts)

A trough is $10\,ft$ long and its ends have the shape of isosceles triangles that are $3\,ft$ across at the top and have a height of $1\,ft$. If the trough is being filled with water at a rate of $12\,ft^3/min$, how fast is the water level rising when the water is 6 inches deep?

_________ ft/min

Answer(s) submitted:

-

(incorrect)

14. (0 pts) A spherical snowball is melting in such a way that its diameter is decreasing at rate of 0.2 cm/min. At what rate is the volume of the snowball decreasing when the diameter is 9 cm.

Your answer _________ (cubic centimeters per minute) should be a positive number.

Hint: The volume of a sphere of radius r is

$$V = \frac{4\pi r^3}{3}.$$

The diameter is twice the radius.

Answer(s) submitted:

-

(incorrect)

15. (0 pts) Water is leaking out of an inverted conical tank at a rate of 14600.0 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 15.0 m and the the diameter at the top is 5.0 m. If the water level is rising at a rate of 18.0 cm/min when the height of the water is 5.0 m, find the rate at which water is being pumped into the tank in cubic centimeters per minute.

Answer: ______________ cm^3/min

Answer(s) submitted:

-

(incorrect)

16. (0 pts) Suppose $xy = -1$ and $\dfrac{dy}{dt} = -2$. Find $\dfrac{dx}{dt}$ when $x = 4$.

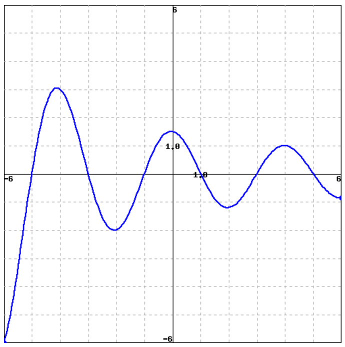
$\dfrac{dx}{dt} =$ _________

Answer(s) submitted:

-

(incorrect)

17. (1 pt) The given graph of the *derivative* f' of a function f is shown. Answer the following questions *only on the interval* $(-6,6)$.



1. On what intervals is f increasing?
Answer (in interval notation): ______

2. On what intervals is f decreasing?
Answer (in interval notation): ______

3. At what values of x does f have a local maximum?
Answer (separate by commas): $x =$ ______

4. At what values of x does f have a local minimum?
Answer (separate by commas): $x =$ ______

Note: *You can click on the graph to enlarge the image.*

Answer(s) submitted:

-
-
-
-

(incorrect)

18. (1 pt) Find the critical numbers of the function

$$f(x) = 2x^3 - 3x^2 - 36x.$$

Answer (separate by commas): $x =$ ______

Answer(s) submitted:

-

(incorrect)

19. (1 pt)

List the critical numbers of the following function separating the values by commas.

$f(x) = 1x^2 + 5x$

______

Answer(s) submitted:

-

(incorrect)

23. (1 pt)

List the critical numbers of the following function in increasing order. Enter N in any blank that you don't need to use.

$s(t) = 3t^4 + 4t^3 - 6t^2$

______

______

______

Answer(s) submitted:

-
-
-

(incorrect)

24. (1 pt)

List the critical numbers of the following function in increasing order. Enter N in any blank that you don't need to use.

$f(z) = \dfrac{6z+6}{8z^2+8z+8}$

______

______

______

Answer(s) submitted:

-
-
-

(incorrect)

$$f(x) = \frac{x^x (x^2-4)^5 (x-1)^3}{e^x \sqrt{x+e^x}}$$

$$\frac{d}{dx} \ln f(x) = \frac{1}{f(x)} f'(x)$$

$$\boxed{f(x) \frac{d}{dx} \ln f(x) = f'(x)}$$

$$\ln f(x) = \ln\left(x^x (x^2-4)^5 (x-1)^3\right) - \ln\left(e^x \sqrt{x+e^x}\right)$$

$$= \ln(x^x) + \ln(x^2-4)^5 + \ln(x-1)^3 - \ln e^x - \ln\sqrt{x+e^x}$$

$$= x\ln x + 5\ln(x^2-4) + 3\ln(x-1) - x - \tfrac{1}{2}\ln(x+e^x)$$

$$\Downarrow \frac{d}{dx}$$

$$\left(x\frac{1}{x} + \ln x\right) + 5\frac{1}{x^2-4}(2x) + 3\frac{1}{x-1}(1) - 1 - \frac{1}{2}\frac{1}{x+e^x}(1+e^x)$$

$$\text{answer} = f'(x) = f(x) \cdot \frac{d}{dx} \ln f(x)$$

$$= \frac{x^x (x^2-4)^5 (x-1)^3}{e^x \sqrt{x+e^x}} \cdot \left(\quad \right)$$

$$\frac{d}{dx}\underbrace{(3x-2)^5(x-1)}_{f(x)}$$

$$\ln f(x) = 5\ln(3x-2) + \ln(x-1)$$

$\downarrow d/dx$

$$\frac{5}{3x-2}(3) + \frac{1}{x-1}$$

$$f'(x) = f(x)\cdot\frac{d}{dx}\ln(f(x))$$

$$= (3x-2)^5(x-1)\left[\frac{5\cdot 3}{3x-2} + \frac{1}{x-1}\right]$$

Using the fact that

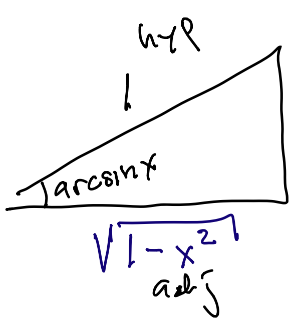
$\sin(\arcsin x) = x$

find $\frac{d}{dx}(\arcsin x)$

$\frac{d}{dx}$ of both sides

$$\cos(\arcsin x)\cdot\frac{d}{dx}\arcsin x = 1$$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\cos(\arcsin x)}$$

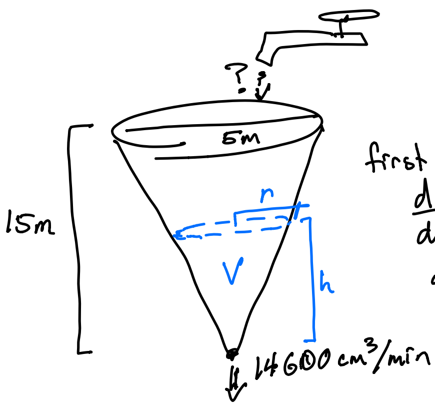


$\arcsin x$ = an angle whose sine is x.

$\Longrightarrow \cos(\arcsin x) = \sqrt{1-x^2}$

$= \text{adj}/\text{hyp}$

$$\frac{d}{dx}\arcsin x = \frac{1}{\cos(\arcsin x)} = \frac{1}{\sqrt{1-x^2}}$$

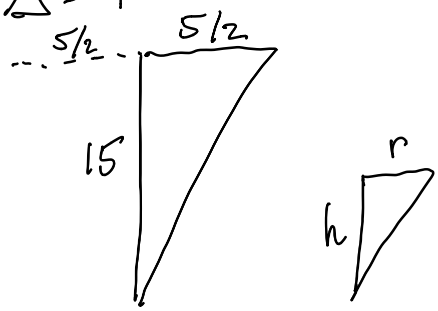


first want $\frac{dV}{dt}$ (when $h = 5m$) and $\frac{dh}{dt} = 18\,\text{cm}/\text{m}$

$$V = \frac{1}{3}\pi r^2 h$$

don't want r.

Similar Δ's to eliminate r



$$\frac{r}{h} = \frac{5/2}{15} = \frac{5}{30} = \frac{1}{6}$$

$$\boxed{r = \frac{h}{6}}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{h}{6}\right)^2 h = \frac{1}{3}\pi\,\frac{h^2}{6^2}\cdot h = \frac{1}{3}\,\frac{1}{6^2}\,\pi h^3$$

$$V = \frac{1}{3}\frac{1}{6^2}\pi h^3$$

$\Downarrow d/dt$

$$\frac{dV}{dt} = \frac{1}{3}\frac{1}{6^2}\pi\, 3h^2 \frac{dh}{dt} = \frac{1}{6^2}\pi h^2 \frac{dh}{dt}$$

to find $\frac{dV}{dt}$, need $\frac{dh}{dt}$ & h

know: $\frac{dh}{dt} = \frac{18}{100}$, $h = 5$.

$\frac{dV}{dt}$ (at the time of intrest)

$$= \frac{1}{6^2}\pi(5)^2\left(\frac{18}{100}\right) \quad m^3/min = 100^3\, cm/min$$

$$= (\text{rate in}) - \underset{14,600}{(\text{rate out})}$$

answer = rate in $= \frac{1}{6^2}\pi(5)^2\left(\frac{18}{100}\right)(100)^3 + \underset{\underset{14,600.}{\shortparallel}}{(\text{rate out})}$

conversion from m^3 to cm^3

A child is carefully inflating a perfectly spherical bubblegum bubble at a rate of 5 cm³/sec. How fast is the surface area changing when the volume is 200 cm³?

- Need an eqn w/ Volume of sphere
- Need an eqn w/ Surface area of sphere.

$$V = \frac{4}{3}\pi r^3 \qquad \boxed{A = 4\pi r^2}$$

~~want~~ have $\frac{dV}{dt}$, don't care about r. want $\frac{dA}{dt}$

strategy: get rid of r from first eqn.

$\underline{V = (\text{Const})\, A^{?}}$

Goal: relate rates of change of V & A

want an eqn w/ V & A.

solve for r in $A = 4\pi r^2$

$$r = \sqrt{\frac{A}{4\pi}} = \frac{A^{1/2}}{2\pi^{1/2}}$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{A^{1/2}}{2\pi^{1/2}}\right)^3 = \frac{4}{3}\pi\,\frac{A^{3/2}}{8\pi^{3/2}}$$

$$V = \frac{4\pi}{3.8\pi^{3/2}} A^{3/2}$$

$$\frac{d}{dt} V = \frac{4\pi}{3.8\pi^{3/2}} \frac{3}{2} A^{1/2} \frac{dA}{dt}$$

want

don't know

but know

$V = 200\ cm^3$

$$\frac{dV}{dt} = 5\ cm^3/sec$$

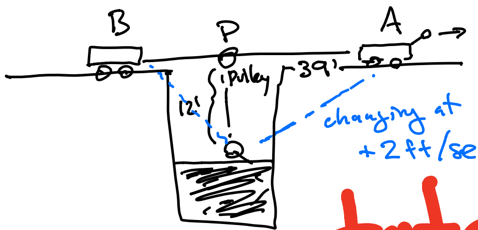
solve for A

$$\frac{3.8\pi^{3/2}}{4\pi} V = A^{3/2}$$

$$\left(\frac{3.8\pi^{3/2}}{4\pi} V\right)^{2/3} = A$$

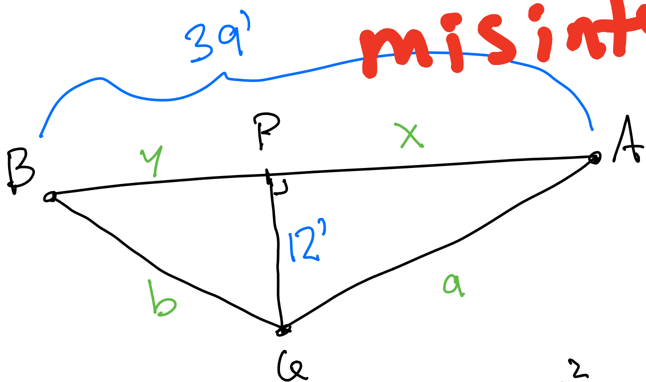
200

$\boxed{\text{HW \#12}}$



WRONG!

total misinterpretation



know $\frac{da}{dt} = 2$

want $\frac{db}{dt}$

when $a = 5$

$12^2 + x^2 = a^2$

$12^2 + y^2 = b^2$

$x + y = 39$

$x = \sqrt{a^2 - 12^2}$

$y = \sqrt{b^2 - 12^2}$

$39 = x + y = \sqrt{a^2 - 12^2} + \sqrt{b^2 - 12^2}$

$39 = \sqrt{a^2 - 12^2} + \sqrt{b^2 - 12^2}$

d/dt both sides

$$0 = \frac{1}{\cancel{2}}(a^2-12^2)^{-1/2} \cdot \cancel{2}a\frac{da}{dt} + \frac{1}{\cancel{2}}(b^2-12^2)^{-1/2}\cancel{2}b\frac{db}{dt}$$

$$\frac{db}{dt} = \frac{-(a^2-12^2)^{-1/2}a\frac{da}{dt}}{(b^2-12^2)^{-1/2}b}$$

$$= \frac{(b^2-12^2)^{1/2}\left(-a\frac{da}{dt}\right)}{(a^2-12^2)^{1/2}b}$$

$$\frac{db}{dt} = \sqrt{\frac{b^2-12^2}{a^2-12^2}}\left(-\frac{a}{b}\right)\frac{da}{dt}$$

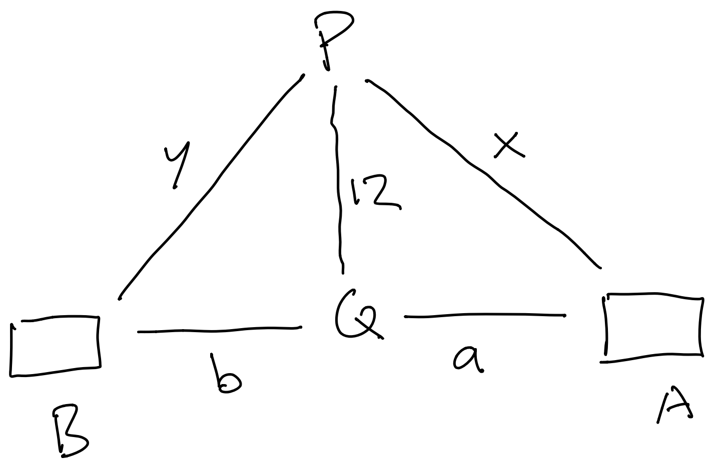
know $a=5$ $\quad \frac{da}{dt}=2$

need b

find $x = \sqrt{a^2-12^2}$ $\qquad y = 39-x$

$= \sqrt{5^2-12^2}$

try again #12



$x + y = 39$

$\frac{da}{dt} = 2$

want $\frac{db}{dt}$ when $a = 5$.

$$\boxed{\begin{array}{cc} a^2 + 12^2 = x^2 & b^2 + 12^2 = y^2 \\ \multicolumn{2}{c}{x + y = 39} \end{array}}$$

$\sqrt{a^2 + 12^3} = x \qquad \sqrt{b^2 + 12^2} = y$

$$\sqrt{a^2 + 12^2} + \sqrt{b^2 + 12^2} = x + y = 39$$

$$\sqrt{a^2 + 12^2} + \sqrt{b^2 + 12^2} = 39$$

$\Downarrow d/dt$

$$\frac{1}{\cancel{2}}(a^2+12^2)^{-1/2}\,\cancel{2}a\frac{da}{dt}+\frac{1}{\cancel{2}}(b^2+12^2)^{-1/2}\,\cancel{2}b\frac{db}{dt}=0$$

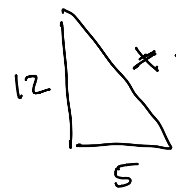
13^2

$$\frac{db}{dt}=\frac{(a^2+12^2)^{-1/2}\left(-a\frac{da}{dt}\right)}{(b^2+12^2)^{-1/2}\,b}$$

26^2

know $\frac{da}{dt}$, a, need to find b.

$\frac{da}{dt} = 2$, $a = 5$



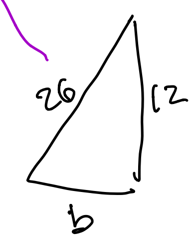
$x=\sqrt{12^2+5^2}$

$=\sqrt{144+25}$

$=\sqrt{169}=13$

$y=39-13$

$=26$



$b=\sqrt{26^2-12^2}$

$=\sqrt{676-144}$

$=\sqrt{532}$

$$x + 3y - \sin(y) = 17$$

Want $\frac{dx}{dt}$ if $\frac{dy}{dt} = 3$ & $y = 0$ $\quad d/dt$

$$\frac{d}{dt}x + \frac{d}{dt}3y - \frac{d}{dt}\sin y = 0$$

$$\underbrace{\frac{dx}{dt}}_{\text{want}} + 3\underbrace{\frac{dy}{dt}}_{\text{know } 3} - \cos \underset{\text{know } 0}{y} \underbrace{\frac{dy}{dt}}_{\text{know } 3} = 0$$

$$\frac{dx}{dt} = -3(3) + \cos 0 \cdot 3$$

$$= -9 + 3 = -6.$$

<u>From last HW</u>

$$\frac{d}{dx}\tan^{-1}(\ln 4x)$$

$$= \frac{1}{(\ln 4x)^2+1} \cdot \underbrace{\frac{d}{dx}\ln 4x}_{\frac{1}{4x}\cdot 4 = \frac{1}{x}}$$

$$y = \tan^{-1}(\ln 4x)$$

$$\tan y = \ln 4x$$

$$\sec^2 y \frac{dy}{dx} = \frac{1}{4x} \cdot 4$$

$$\frac{dy}{dx} = \frac{\cancel{4}}{\cancel{4}x} \frac{1}{\sec^2 y}$$

$$\frac{dy}{dx} = \frac{1}{x}\cos^2 y$$

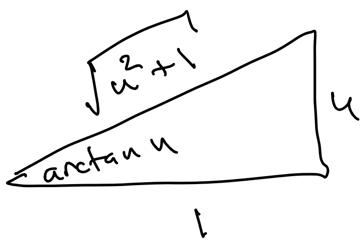
remember:

$\cos(\tan^{-1} u)$

$$= \frac{1}{x}\cos^2(\tan^{-1}(\ln 4x))$$

$\tan^{-1} u = \arctan u$

is an angle whose tangent is u.



$\cos(\tan^{-1} u)$

$$= \frac{1}{\sqrt{u^2+1}}$$

"simplify"

$$\frac{dy}{dx} = \frac{1}{x}\,\frac{1}{(\ln 4x)^2+1}$$

Other way

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{x^2+1}$$

$$\tan(\tan^{-1} x) = x$$

$$\Downarrow d/dx$$

$$\sec^2(\tan^{-1}x)\cdot \underbrace{\frac{d}{dx}\tan^{-1} x}_{\text{solve}} = 1$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{\sec^2(\tan^{-1}x)} = \cos^2(\tan^{-1}x)$$

$$\cos^2(\tan^{-1}x) = \left(\frac{1}{\sqrt{x^2+1}}\right)^2 \quad (\Delta\text{'s})$$

$$\boxed{\frac{d}{dx}\tan^{-1}x = \frac{1}{x^2+1}}$$