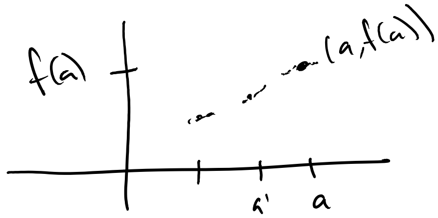
Lecture 4: One-sided limits and continuity, slopes and tangent lines

Thursday, January 19, 2017 12:31 PM

How to do well in the class

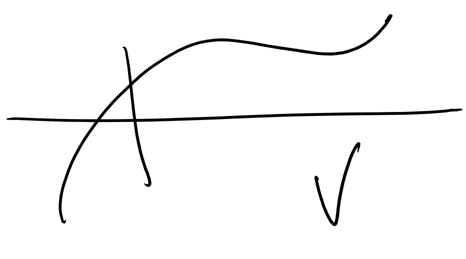
- make sure you understand everything.
- resolve all uncertainties → email
  → office hours
  → webwork: email instructor
  → ask questions in class.
- successfull completion of webwork ≠ understanding.

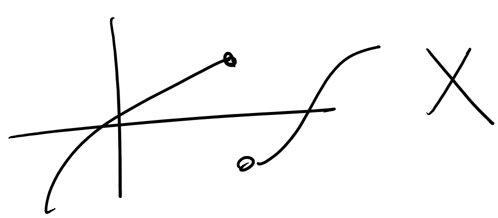
Visual interpretation of continuity:



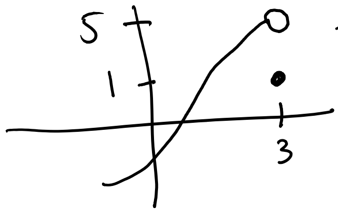
$\lim_{x \to a} f(x) = f(a)$

→ can draw graph "without lifting your pen"





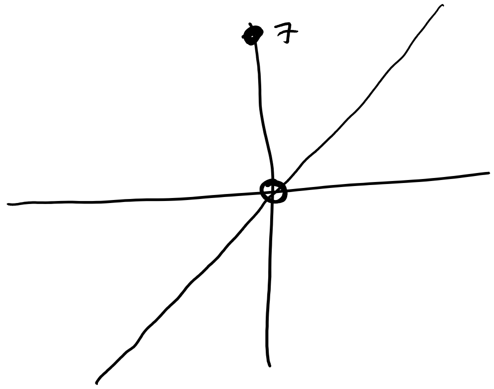
Notation



$f(x)$

$f(3) = 1$

$$f(x) = \begin{cases} x & \text{if } x \neq 0 \\ 7 & \text{if } x = 0 \end{cases}$$



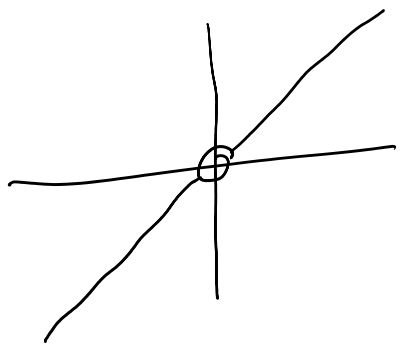
for above example

$$\lim_{x \to 0} f(x) = 0 \neq f(0) = 7$$

how? $\lim_{x \to 0} f(x) = \lim_{x \to 0} x$

since $x \neq 0$

$$f(x) = \frac{x^2}{x}$$



$$\lim_{x \to 0} f(x) = 0$$

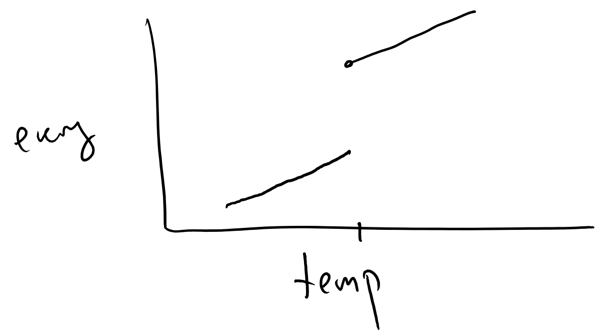
$\frac{x^2}{x} = x$

if $x \neq 0$.

My <u>favorite discontinuous function</u>

fixed mass of water at const. pressure adding energy (heat)

temerature vs energy



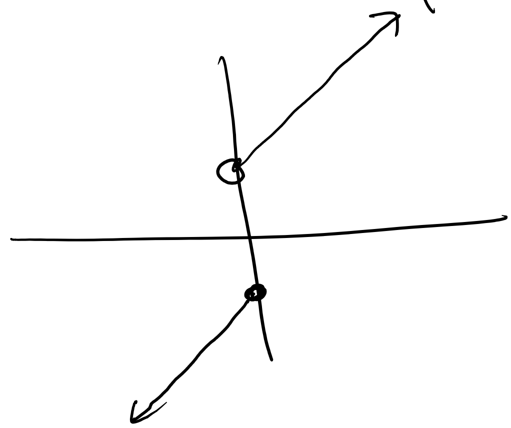
Caution: steam

$\lim_{x \to 0} f(x) =$ doesn't exist.

$f(x) = \begin{cases} x+1 & \text{if } x > 0 \\ x-1 & \text{if } x \le 0 \end{cases}$

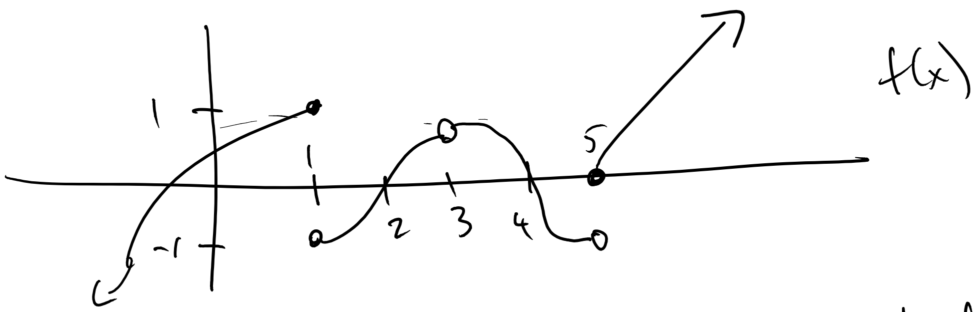
$\lim_{x \to 0^+} f(x) = 1$

$\lim_{x \to 0^-} f(x) = -1$



Definition $\lim_{x \to a^+} f(x) = L$ means that we can make $f(x)$ as close as we want to L by making x close to, but strictly greater than a.

$\lim_{x \to a^-} f(x) = L$ means that we can make $f(x)$ as close as we want to L by making x close to, but strictly less than a.



1. $\lim\limits_{x\to 1} f(x)$ = d.n.e.

2. $\lim\limits_{x\to 1^-} f(x)$ = 1

3. $\lim\limits_{x\to 2^-} f(x)$ = 0

4. $\lim\limits_{x\to 3} f(x) = 1$

5. $\lim\limits_{x\to 5^+} f(x) = 0$

6. Domain of $f(x)$ (interval notation)

$(-\infty,3)\cup(3,\infty)$

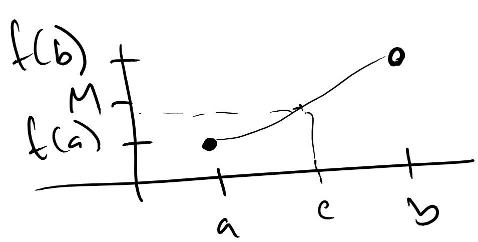
7. Where is $f(x)$ continuous?

$(-\infty,1)\cup(1,3)\cup(3,5)\cup(5,\infty)$

not cont: $\{1,3,5\}$

Most important continuity fact:

<u>Intermediate value theorem</u>

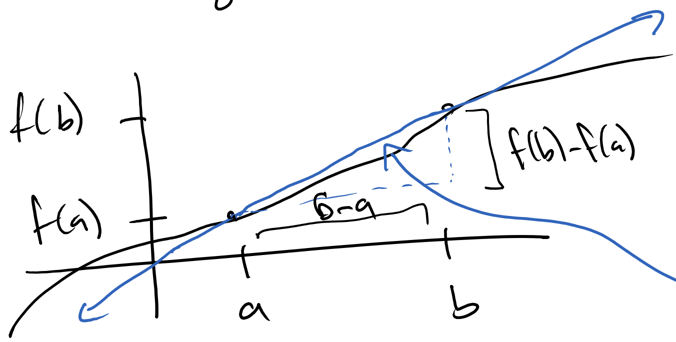


if $a<b$ & $f(a)<f(b)$ then for any $f(a)<M<f(b)$ there is some $a<c<b$ w/ $f(c)=M$.

Beginnings of derivatives/rates of change.

Starting point: Average rates of change.

Given a function $f(x)$, ask how quickly on average does f change w/ respect to x between $x=a$ & $x=b$?

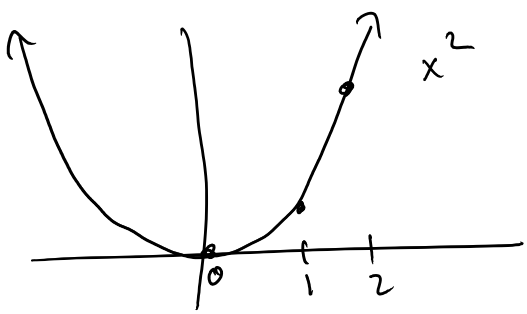


answer $= \dfrac{\text{total change in } y}{\text{total change in } x}$

$= \dfrac{f(b)-f(a)}{b-a}$

slope of secant line

ex: $f(x)=x^2$ av. rate of chg between 1 & 2



$\dfrac{f(2)-f(1)}{2-1} = \dfrac{4-1}{1} = 3.$

what if we want to know slope at $x=1$

closer: avg. rate of chg between 1 & 1.1

$$\frac{f(1.1)-f(1)}{1.1-1} = \frac{1.21-1}{.1} = \frac{.21}{.1} = 2.1$$

between 1 & 1.001 ...

between 1 & $1+h$ h small

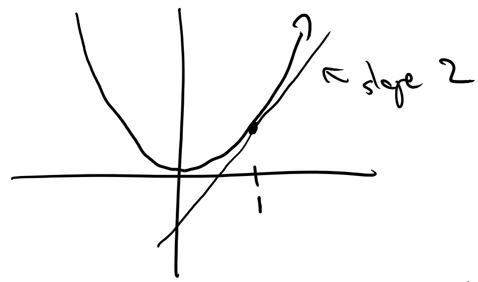
$$\frac{f(1+h)-f(1)}{1+h-1} = \frac{1+2h+h^2-1}{h}$$

$$\frac{\quad}{1+h-1} \quad - \quad \frac{\quad}{h}$$

$$= \frac{2h+h^2}{h} = 2+h$$

as h gets smaller, this gets close to 2

$\lim\limits_{h\to 0} 2+h = 2$ slope at $x=1$ is $\underline{2}$



<u>Def</u> the average rate of change of $f(x)$ between a & b is

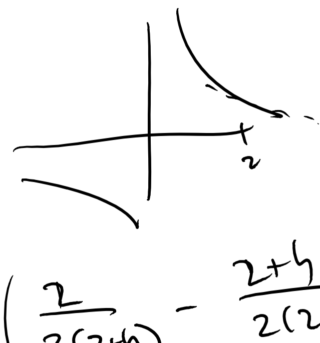
$$\frac{f(b)-f(a)}{b-a}$$

<u>Def</u> the instantaneous rate of change of $f(x)$ at $x=a$

is $\lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{a+h-a} = \lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$

this is also called "the derivative of $f(x)$ at $x=a$"

<u>ex:</u> inst. rate of chge of $f(x) = 1/x$ at $x=2$.



$$\lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \lim_{h\to 0} \frac{\left(\frac{2}{2(2+h)} - \frac{2+h}{2(2+h)}\right)}{h}$$

h

$$= \lim_{h \to 0} \frac{2-(2+h)}{2(2+h)h} = \lim_{h \to 0} \frac{-\cancel{h}}{2(2+h)\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-1}{2(2+h)} = -\frac{1}{4}$$