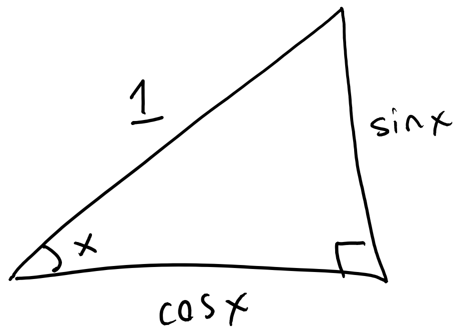
Lecture 6: various things (infinite limits, trigonometric limits, asymptotes)

Tuesday, January 24, 2017 12:35 PM

(sec. 2.2)

Trig reminder



$$\sin^2 x + \cos^2 x = 1$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

div. by $\sin^2 x$

$$1 + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$1 + \cot^2 x = \csc^2 x$$

div by $\cos^2 x$

$$\frac{\sin^2 x}{\cos^2 x} + 1 = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

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infinite limits

We say that $\lim_{x\to a} f(x) = \infty$ if we can make $f(x)$ as large as we want by making x sufficiently close to a.

Similarly $\lim_{x\to a} f(x) = -\infty$ if - - - - $f(x)$ as [illegible] negative as we want - - — — — a.

largely negative as we want - — —— ———

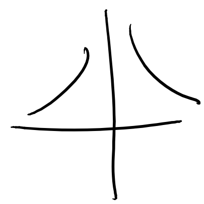
Similarly, $\lim_{x\to a^+} f(x) = \infty$ or $\lim_{x\to a^-} f(x) = \infty$ or

$\lim_{x\to a^+} f(x) = -\infty$ or . .

<u>examples</u> $\lim_{x\to 0^+} \frac{1}{x} = \infty$ (if x is small, positive, $\frac{1}{x}$ big positive)

$\lim_{x\to 0^-} \frac{1}{x} = -\infty$

$\lim_{x\to 0} \frac{1}{x} =$ d.n.e. $\quad \lim_{x\to 0^+} \frac{1}{x^2} = \infty \quad \lim_{x\to 0} \frac{1}{x^2} = \infty$



<u>Note:</u> $\lim_{x\to a} f(x) = M$ means $\lim_{x\to a^-} f(x) = M$ & $\lim_{x\to a^+} f(x) = M$ (still even if $M = \pm\infty$)

How to find infinite limits:

If $f(x) = \frac{g(x)}{h(x)}$ with $\lim_{x\to a^+} g(x) = L > 0$ and

$\lim_{x\to a^+} h(x) = 0$ and $h(x) > 0$ for x close to, and larger than a

then $\lim_{x\to a^+} f(x) = \infty$

then $\lim_{x \to a^+} f(x) = \infty$

ie: $\frac{\text{positive}}{+\text{small}} = +\text{big}$.

$\frac{\text{neg}}{+\text{small}} = -\text{big}$.

$\frac{\text{neg}}{-\text{big}} = +\text{small}$

$$\lim_{x \to 5^+} \frac{x+2}{x-5} = \infty$$

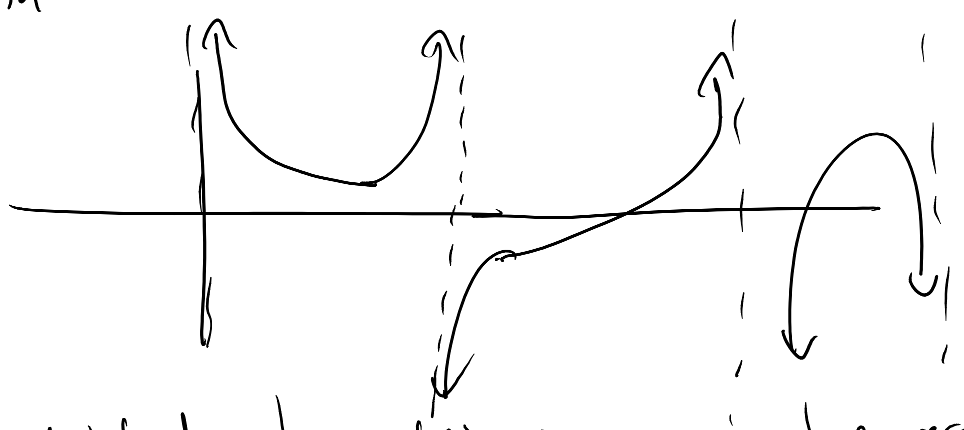
$$\frac{7\,(\text{pos})}{+\text{small}} = +\text{big}$$

ex: graph $\frac{(x-1)(x+3)}{(x+2)(x-5)} = f(x)$

1. when above, when below x axis?
2. what are the vertical asymptotes.

Def we say that $f(x)$ has a v. asymptote at $x=a$ if

$\lim_{x \to a^+} f(x) = \pm\infty$ or $\lim_{x \to a^-} f(x) = \pm\infty$



we want to first find where $f(x)$ is pos & where neg

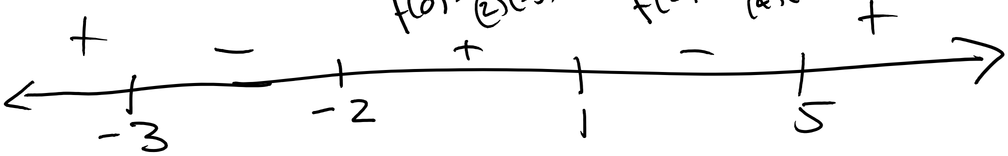
$$f(x) = \frac{(x-1)(x+3)}{(x+2)(x-5)}$$

IVT $\Rightarrow$ f can only change signs by passing through 0 or by being discontinuous.

where is $f(x) = 0$? ← $x = 1, -3$

where is $f(x)$ discontinuous? ← $x = -2, 5$

$f(0) = \frac{(-1)(3)}{(2)(-5)}$ $f(2) = \frac{(1)(5)}{(4)(-3)}$



f(x) has same sign consistently within each of these regions.

To find asymptotes:
notice: can only happen when f(x) is discontinuous.
only possibilities are $-2, 5$

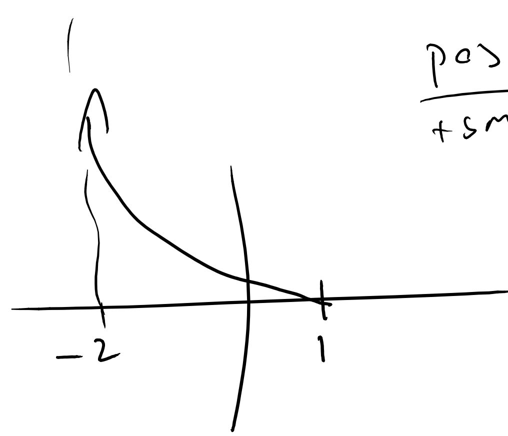
$$\lim_{x\to -2^+} \frac{(x-1)(x+3)}{(x+2)(x-5)} = \infty$$

top $\approx (-2-1)(-2+3)$ neg

bot $\approx$ +small $\cdot (-7)$

$\frac{\text{pos}}{\text{+small}}$ = +big

$$\lim_{x\to 2^-} \frac{(x-1)(x+3)}{(x+2)(x-5)}$$



$$\lim_{x\to 5^+}$$