Lecture 7: Derivatives, full on

Monday, January 30, 2017 12:25 PM

Ex: $f(x) = 3x^2+1$ $\quad f'(2) = \lim\limits_{h\to 0} \dfrac{f(2+h)-f(2)}{h} = \lim\limits_{h\to 0} \dfrac{(3(2+h)^2+1)-(3(2)^2+1)}{h}$

or gen. formula $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{(3(x+h)^2+1)-(3x^2+1)}{h}$

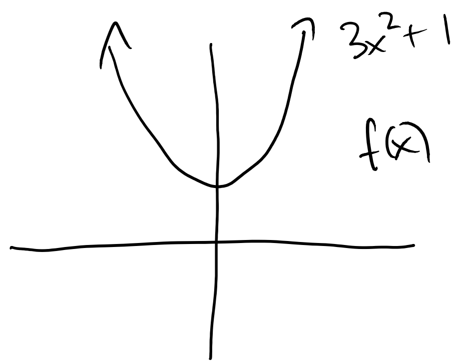
$$= \lim_{h\to 0} \frac{(3(x^2+2xh+h^2)+1)-(3x^2+1)}{h}$$

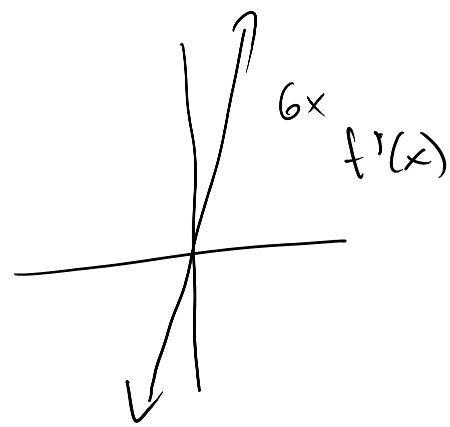
$$= \lim_{h\to 0} \frac{\cancel{3x^2}+6xh+3h^2+\cancel{1}-\cancel{3x^2}-\cancel{1}}{h}$$

$$= \lim_{h\to 0} \frac{6xh+3h^2}{h} = \lim_{h\to 0} 6x+3h = 6x$$

$f'(2) = 6(2) = 12$

Visually:





Notation: $y = f(x)$ $\quad$ **Derivative:** $f' = f'(x) = \dfrac{dy}{dx} = \dfrac{d}{dx}y$

$$= \frac{d}{dx}f = \frac{d}{dx}f(x) = \frac{df}{dx} = y'$$

$f(x) = 3x^2+1$ $\quad f'(x) = 6x$

$(3x^2+1)' = 6x$ $\quad \dfrac{d}{dx}(3x^2+1) = 6x$

Differentiation Rules (3.2), (5)

Sum / Difference / Const. mult rule

$$\frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx} \qquad \frac{d}{dx}(Cf) = C\frac{df}{dx}$$

$$\frac{d}{dx}(f-g) = \frac{df}{dx} - \frac{dg}{dx}$$

Why?

$$\frac{d}{dx}(f-g) = \lim_{h\to 0} \frac{(f-g)(x+h) - (f-g)(x)}{h}$$

$$= \lim_{h\to 0} \frac{\big(f(x+h) - g(x+h)\big) - \big(f(x) - g(x)\big)}{h}$$

$$= \lim_{h\to 0} \frac{\big(f(x+h) - f(x)\big) - \big(g(x+h) - g(x)\big)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} - \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

$$= f'(x) - g'(x)$$

Product Rule

$$\frac{d}{dx}(fg) = f'g + fg' \qquad \text{(Liebnitz Rule)}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf' - fg'}{g^2} \qquad \frac{d}{dx}\left(\frac{HI}{LO}\right) = \frac{LO\,dHI - HI\,dLO}{LO\,LO}$$

$$\frac{d}{dx}\left(f/g\right) = \frac{gf' - f'g}{g^2}$$

$$\frac{d}{dx}(\text{[illegible]}) = -\,\text{[illegible]}$$

Power Rule

$$\frac{d}{dx}(x^n) = \lim_{h\to 0}\frac{(x+h)^n - x^n}{h} = \lim_{h\to 0}\frac{\cancel{x^n} + nhx^{n-1} + (\text{stuff})h^2 - \cancel{x^n}}{h}$$

$$= \lim_{h\to 0}\frac{nhx^{n-1} + (\text{stuff})h^2}{h} = \lim_{h\to 0} nx^{n-1} + (\text{stuff})h$$

$$= nx^{n-1}$$

$$\boxed{\frac{d}{dx}(nx^{n-1})}$$

$$\frac{d}{dx}(c) = 0$$

Examples

$$\frac{d}{dx}(3x^2+1) = \frac{d}{dx}(3x^2) + \frac{d}{dx}(1)$$

$$= 3\frac{d}{dx}(x^2) + \frac{d}{dx}(1)$$

$$= 3(2x^1) + 0 = 6x$$

$$\frac{d}{dx}\left(\frac{x+1}{x-1}\right) = \frac{(x-1)(x+1)' - (x+1)(x-1)'}{(x-1)^2}$$

$$= \frac{(x-1)\left[(x)' + (1)'\right] - (x+1)\left[(x)' - (1)'\right]}{(x-1)^2}$$

$$\frac{d}{dx}(x) = \frac{d}{dx}(x^1) = 1\cdot x^0 = 1 \qquad \frac{d}{dx}(1) = 0$$

$$= \frac{(x-1)(1)-(x+1)(1)}{(x-1)^2} = \frac{x-1-x-1}{(x-1)^2}$$

$$= \frac{-2}{(x-1)^2}$$

$$\frac{d}{dx}\left[(3x+1)(4x^3-2)\right] = (3x+1)'(4x^3-2) + (3x+1)(4x^3-2)'$$

$$\frac{d}{dx}\left(\frac{3x^2+2x}{x}\right) \longrightarrow \frac{d}{dx}(3x+2)$$

$$\frac{d}{dx}(3x^2+2x)(x^{-1})$$

$$\frac{d}{dx}\left(\frac{1}{x+1}\right)$$

$$\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{(x^n)(1)'-1(x^n)'}{(x^n)^2}$$

$$= \frac{-(x^n)'}{x^{2n}}$$

$$= \frac{-nx^{n-1}}{x^{2n}}$$

$$= -nx^{n-1}x^{-2n}$$

$$= -nx^{n-1-2n}$$

$$= -nx^{-1-n}$$

$$= -nx^{-n-1}$$

$$\frac{d}{dx}x^{\pi} = \pi x^{\pi-1}$$

$$\frac{d}{dx}(x^{-n})$$