Lecture 8: derivatives, derivatives, derivatives

Tuesday, January 31, 2017 12:37 PM

(Section 4)

$$\lim_{h\to 0} \frac{\sin h}{h} = 1$$

$\sin(A+B) = \sin A \cos B + \sin B \cos A$

(remember: $e^{i\theta} = \cos\theta + i\sin\theta$)
Euler formula

$$\frac{d}{dx}\sin x = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x\cosh + \sin h\cos x - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x\cos h - \sin x}{h} + \frac{\sin h\cos x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x\cos h - \sin x}{h} + \lim_{h\to 0} \frac{\sin h\cos x}{h}$$

$$= \sin x \underbrace{\lim_{h\to 0} \frac{\cos h - 1}{h}}_{0} + \cos x \underbrace{\lim_{h\to 0} \frac{\sin h}{h}}_{1} = \cos x$$

$$\lim_{h\to 0} \frac{\cos h - 1}{h} = \lim_{h\to 0} \left(\frac{\cos h - 1}{h}\right)\left(\frac{\cos h + 1}{\cos h + 1}\right)$$

$$\boxed{\begin{aligned} &\cos^2 h + \sin^2 h = 1 \\ &\cos^2 h - 1 = -\sin^2 h \end{aligned}}$$

$$= \lim_{h\to 0} \frac{\cos^2 h - 1}{h(\cos h + 1)} = \lim_{h\to 0} \frac{-\sin^2 h}{h(\cos h + 1)}$$

$$\lim_{h\to 0} \left(\frac{\sin h}{h}\right)\left(\frac{-\sin h}{\cos h + 1}\right) = \underbrace{\lim_{h\to 0} \left(\frac{\sin h}{h}\right)}_{1} \underbrace{\lim_{h\to 0} \frac{-\sin h}{(\cos h + 1)}}_{\text{(plug in)}\ \frac{-0}{(1+1)} = 0} = 0$$

$h \to 0$

1 (plug in) $\frac{}{(1+1)} = 0$

$1 \cdot 0$

$\cos(A+B) =$

<u>So:</u> $\frac{d}{dx} \sin x = \cos x$

similarly could do

$$\frac{d}{dx} \cos x = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h} = \lim_{h \to 0} \frac{\cos x \cosh - \sin x \sin h - \cos x}{h}$$

$$\ldots\ldots = -\sin x$$

$\frac{d}{dx} \cos x = -\sin x$ $\qquad$ $\frac{d}{dx} \sin x = \cos x$

$$\frac{d}{dx} \tan x = \frac{d}{dx} \frac{\sin x}{\cos x} \overset{\text{quot rule}}{=} \frac{1}{\cos^2 x} = \sec^2 x$$

(Theorem 4.6.5)

$\frac{d}{dx} \sec x = \frac{d}{dx} \frac{1}{\cos x} = \sec x \tan x$ $\left(\frac{\sin x}{\cos^2 x}\right)$

$\frac{d}{dx} \csc x = \frac{d}{dx} \frac{1}{\sin x} = -\csc x \cot x$

$\frac{d}{dx} \cot x = -\csc x$

$$\frac{d}{dx}(3x^2-\sin x)(1+\cos x)$$

$$= (3x^2-\sin x)'(1+\cos x) + (3x^2-\sin x)(1+\cos x)'$$

$$= (3\cdot 2x-\cos x)(1+\cos x) + (3x^2-\sin x)(-\sin x)$$

$f(x)=e^x$ $\qquad$ $e=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

$$a^{x+y}=a^x a^y$$

$$\frac{d}{dx}a^x = \lim_{h\to 0}\frac{a^{x+h}-a^x}{h} = \lim_{h\to 0}\frac{a^x a^h - a^x}{h}$$

$$= a^x \lim_{h\to 0}\frac{a^h-1}{h}$$

turns out, if $a=e$ then $\lim_{h\to 0}\frac{e^h-1}{h}=1$

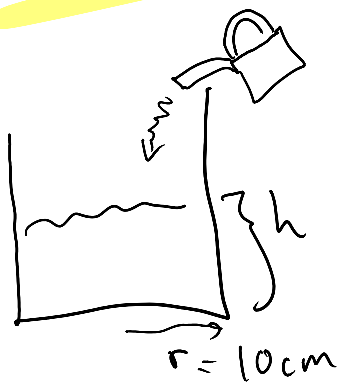
$$\Longrightarrow \frac{d}{dx}e^x = e^x(1)=e^x$$

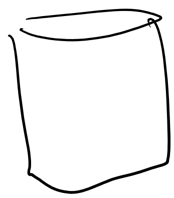
Ex: $\frac{d}{dx}e^x\sin x = (e^x)'\sin x + e^x(\sin x)'$

$$= e^x\sin x + e^x\cos x$$

$= e^x(\sin x + \cos x)$

Chain Rule





pouring at rate of
$30\ \text{cm}^3/\text{sec}$

how fast is
water rising?

$\frac{dV}{dt}$ volume per time

want height / time $\frac{dh}{dt}$

$V = \pi r^2 h = 100\pi h$ $\qquad \frac{dh}{dV}$

$h =$ function of V

(out of time :( )