Lecture 9: Chain rule

Thursday, February 2, 2017 12:34 PM

Caffe hour tomorrow 12:30-2 in "the Matrix" (Boyd 308)

Differentiation rules so far:

$$\frac{d}{dx}x^n = nx^{n-1} \qquad \frac{d}{dx}e^x = e^x \qquad \frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$(f+g)' = f' + g', \quad (fg)' = f'g + fg', \quad \left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

$$(Cf)' = Cf'$$

$$\frac{d}{dx}\frac{x^2e^x - x^2}{x - \sin x} = \frac{(x-\sin x)\frac{d}{dx}(x^2e^x - x^2) - (x^2e^x - x^2)\frac{d}{dx}(x - \sin x)}{(x-\sin x)^2}$$

$$= \frac{(x-\sin x)\left(\frac{d}{dx}x^2e^x - \frac{d}{dx}x^2\right) - (x^2e^x - x^2)\left(\frac{d}{dx}(x) - \frac{d}{dx}\sin x\right)}{(x-\sin x)^2}$$

$$= \frac{(x-\sin x)\left(2xe^x + x^2e^x - 2x\right) - (x^2e^x - x^2)(1 - \cos x)}{(x-\sin x)^2}$$

$$\frac{d}{dx} \sin x \cos x\, e^x = \frac{d}{dx}\left[(\sin x \cos x)\, e^x\right]$$

$$= (\sin x \cos x)' e^x + (\sin x \cos x)(e^x)'$$

$$= \left((\sin x)' \cos x + \sin x (\cos x)'\right) e^x + (\sin x \cos x) e^x$$

$$= \left(\cos^2 x - \sin^2 x\right) e^x + (\sin x \cos x) e^x$$

Chain Rule

given: dollars/hour $= \$14$, cookies/dollar $= 2$,

cookies/hour = (dollars/hour)(cookies/dollar) = 28

C = cookies D = dollars H = hours

rate of chg of dollars w/ resp. to hours

$D(H)$ $\quad$ $C(D)$

$$\frac{dC}{dH} = \frac{dC}{dD}\frac{dD}{dH}$$

rate of change of cookies with resp. to dollars

$C(D) = D^2$

ex: $C(D) = D^2$

$D(H) = e^H - H^2 + 100$

suppose work 3 hours want $\frac{dC}{dH}$? when $H=3$

$$\frac{dD}{dH} = \frac{d}{dH} D = \frac{d}{dH}\left(e^H - H^2 + 100\right) = e^H - 2H$$

$$\frac{dC}{dD} = \frac{d}{dD}(C) = \frac{d}{dD}(D^2) = 2D$$

$$\frac{dC}{dH} = \frac{dC}{dD}\frac{dD}{dH} = (2D)(e^H - 2H)$$

plug in $H=3$.

$$D = e^H - H^2 + 100 \rightsquigarrow 2D = 2(e^H - H^2 + 100)$$

$$\left.\frac{dC}{dH}\right|_{H=3} = \left.2D\right|_{H=3}\left.(e^H - 2H)\right|_{H=3}$$

$$= 2D(3)\,e^3 - 2(3)$$

$$\frac{d}{dH} C(D(H)) = \frac{dC}{dD}\frac{dD}{dH}$$

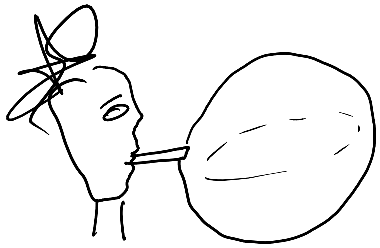
$f = C = D^2$

$f' = 2D$

$$\boxed{\frac{d}{dx} f(g(x)) = f'(g(x))\,g'(x)}$$

$\frac{d}{dx} f(g(x))$

$= \frac{df}{dg} \frac{dg}{dx}$



inflating at a rate of $70 \text{ cm}^3/\text{sec}$
how fast is the surface area changing
when the volume is 400 cm^3

$$\frac{dS}{dt} = \frac{dS}{dV} \cdot \underbrace{\frac{dV}{dt}}_{70}$$

know $\frac{dV}{dt}$

$$S = 4\pi r^2 = 4\pi \left(\frac{3V}{4\pi}\right)^{2/3}$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{3}{4\pi} V = r^3$$

$$\left(\frac{3}{4\pi} V\right)^{1/3} = r$$

$S(V)$

$$= 4\pi \left(\frac{3V}{4\pi}\right)^{2/3} = 4\pi \left(\frac{3}{4\pi}\right)^{2/3} V^{2/3}$$

$$\frac{dS}{dV} = 4\pi \left(\frac{3}{4\pi}\right)^{2/3} \frac{2}{3} V^{-1/3}$$

$$\frac{dS}{dt} = 4\pi \left(\frac{3}{4\pi}\right)^{2/3} \cdot \frac{2}{3} V^{-1/3} \cdot 70$$

if $V = 400$

$$= 4\pi \left(\frac{3}{4\pi}\right)^{2/3} \cdot \frac{2}{3} \frac{1}{\sqrt[3]{400}}$$

$\left(3\right)^{2/3} \quad 2 \quad 2$

$\sim 12\,(12)\ {}^{'}\bar{3}\ \ 15$

$\sim 12\left(\frac{1}{4}\right)^{2/3} \cdot \frac{2}{3}\,\frac{2}{15} = 12 \cdot \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} \cdot \frac{2}{15} \simeq \frac{1}{2}$ ish