MATH 2250 PRACTICE SHEET FOR FINAL EXAM

1. Use the definition of the derivative to find the derivative of the function

$$f(x) = x^2 + \frac{1}{x}$$

2. Find an equation for the tangent line to the graph of the function

$$f(x) = 3x + \ln x$$

at $x = 1$.

Use this information to approximate $f(1.2)$.

3. Find the derivative of the function

$$f(x) = \frac{xe^x - 1}{\ln x} = \frac{(\ln x)(xe^x + e^x) - (xe^x - 1)\frac{1}{x}}{(\ln x)^2}$$

4. Solve for $\frac{dx}{dt}$ given the equation

$$\ln(x + y) = e^x - t$$

5. Compute the following limit

$$\lim_{x \to 0} \frac{\sin x^2}{\cos x - 1}$$

6. Compute the following limit

$$\lim_{x \to 3} \frac{e^x - e^3}{x - 3}$$

7. Compute the following limit

$$\lim_{x \to 3} \frac{e^x - e^3}{x}$$

8. Find the absolute minimum and maximum values of the function $f(x) = x + \ln x$ on the interval $[1, e]$.

9. Does the following function have an absolute maximum or absolute minimum value on the interval $[\frac{1}{2}, \infty)$?

$$f(x) = x - 7\ln x$$

10. Consider the function $f(x) = \dfrac{3}{1+x^3}$, and suppose that $F(x)$ is an antiderivative for $f(x)$ with $F(0) = 0$.

Explain why $F(x) = \displaystyle\int_0^x \frac{3}{1+t^3} dt$

11. Two people start walking from the same point, person A walking due north and person B walking due east. After some time, if person A is 40 feet from the starting point and walking at 3 feet per second, and if person B is 30 feet from the starting point and walking at 5 feet per second, how fast is the distance between the two people changing?

12. Compute

$$\int e^x \cos e^x dx$$

13. Compute

$$\int (\sin x)^7 (\cos x) dx$$

14. Compute

$$\int_0^1 x\sqrt{1-x^2}\ dx$$

15. Compute

$$\int_0^1 \sqrt{1-x^2}\ dx$$

hint: this is a trick question

16. Compute

$$\int \tan x\ dx$$

(you shouldn't need to memorize this formula — use u-substitution!)

17. Find two number a and b such that $3a + 4b = 9$ and such that ab is as large as possible.

18. Find all critical values of the following functions $x, x^{-1}, x^2, x^3, x^{2/3}, x^{-2/3}, x + \ln x$. Which of these critical values represent local minimums and which represent local maximums?

19. Use Riemann Sums with 3 rectangles and using left endpoints to approximate the value of the integral:

$$\int_0^1 \frac{1}{1+x^3}\, dx$$

20. Use Riemann Sums and limits to find the value of the definite integral:

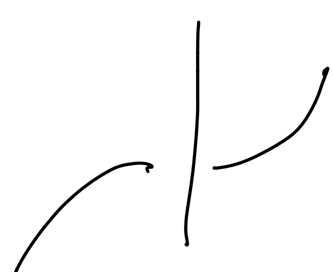
$$\int_2^3 (3x+2)\ dx$$

21. A company would like to design a box (bottom, top and four sides), with square base with a volume of exactly 1000 cubic centimeters. How tall should the box be made so that it uses the least amount of material (surface area)?

22. Suppose that $f(x)$ is defined on $[-3, 3]$ which satisfies the following properties:
- $f(x)$ is increasing on the interval $[-3, 0]$,
- $f(x)$ is decreasing on $[0, 3]$,
- $f(x)$ is concave down on $[-3, 1]$, and
- $f(x)$ is concave up on $[1, 3]$.

Use this information to sketch the graph of $f(x)$.

23. Sketch a graph of a function which is increasing everywhere, concave down for $x < 0$ and concave up for $x > 0$.



on

$$\ln(x+y) = e^x - t \qquad \frac{dx}{dt}$$

$\downarrow d/dt$

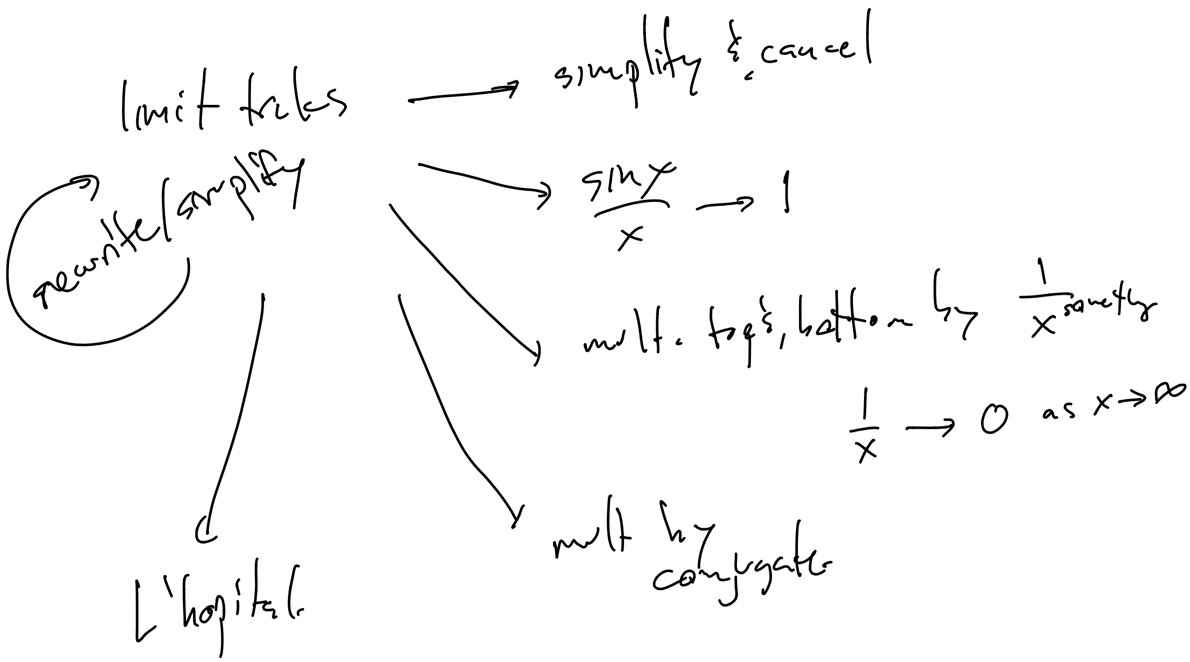
$$\left(\frac{1}{x+y}\right)\left(\frac{dx}{dt}+\frac{dy}{dt}\right) = e^x\frac{dx}{dt} - 1$$

$$\left(\frac{1}{x+y}\right)\frac{dx}{dt} + \left(\frac{1}{x+y}\right)\frac{dy}{dt} = e^x\frac{dx}{dt} - 1$$

$$\left(\frac{1}{x+y}\right)\frac{dx}{dt} - e^x\frac{dx}{dt} = -1 - \frac{1}{x+y}$$

$$\frac{dx}{dt}\left[\frac{1}{x+y} - e^x\right] = -1 - \frac{1}{x+y}$$

$$\frac{dx}{dt} = \frac{\left(-1 - \frac{1}{x+y}\right)}{\left(\frac{1}{x+y} - e^x\right)}$$

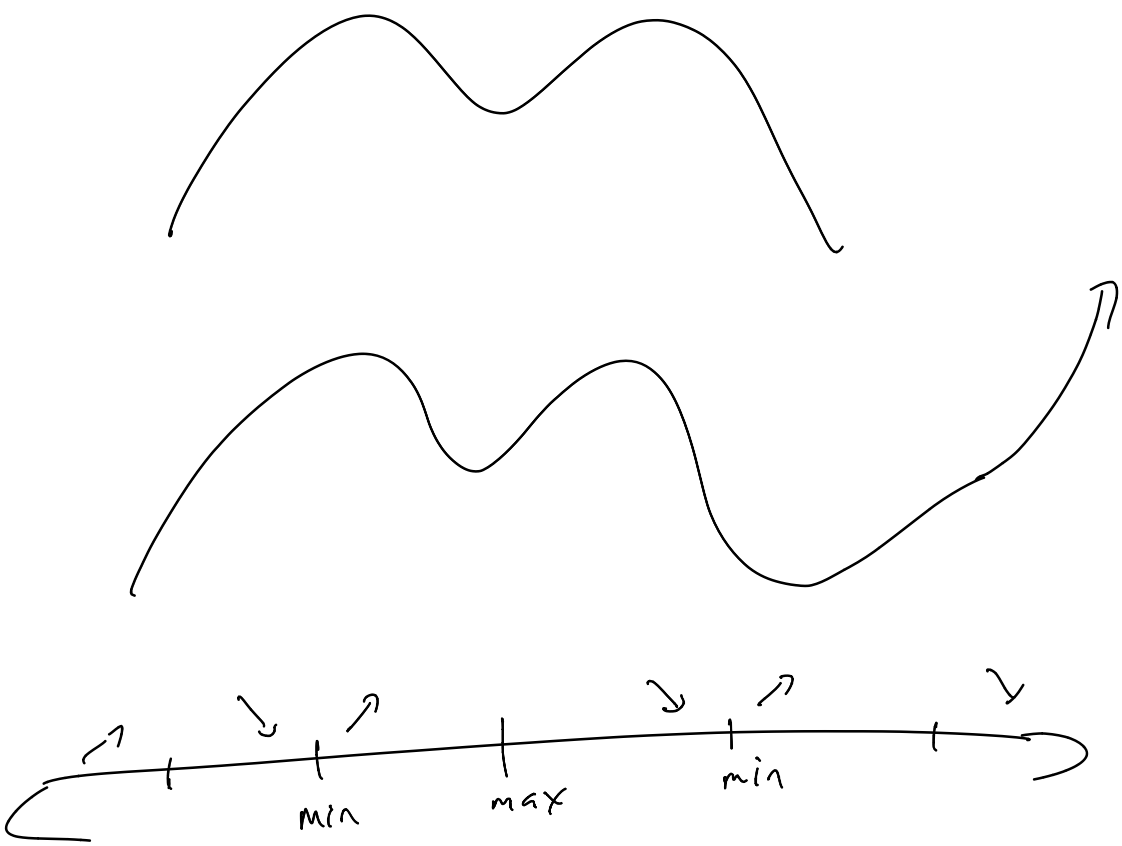


Find two number a and b such that $3a + 4b = 9$ and such that ab is as large as possible.

get rid of a variable

$$f = ab \qquad b = \frac{9-3a}{4}$$

$$f(a) = a\left(\frac{9-3a}{4}\right)$$



min
max
min

$x^2+y^2+z^2+w^2$ 4 squares theorem

x, y integers

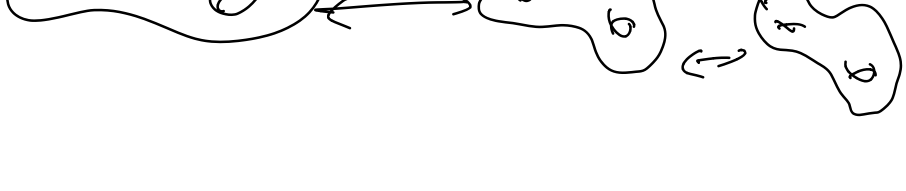
$x^2+3y^2+4z^3+7w^2$

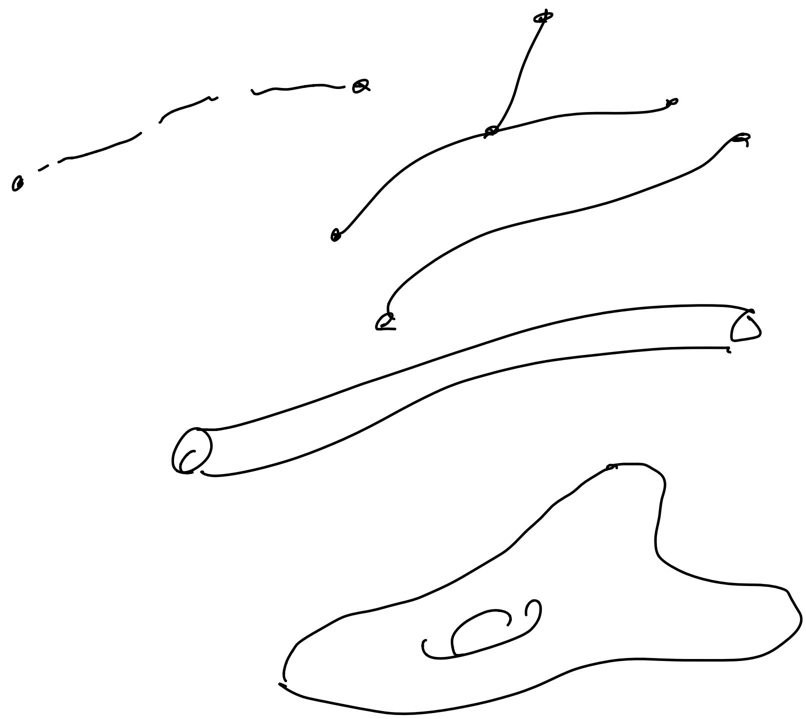
$2x^2+2y^2+2z^2+2w^2$

$9x^2-13y^2+2z^2+7w^2 = 32/5$

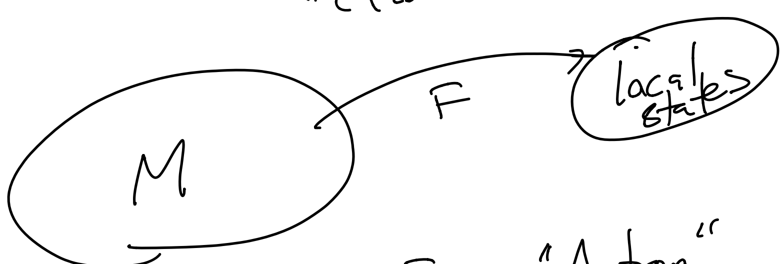
9 17 8

$1+2^3+2^7+2^9+2^{13}+2^{17}$



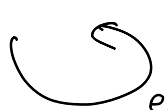


"classical field thy"

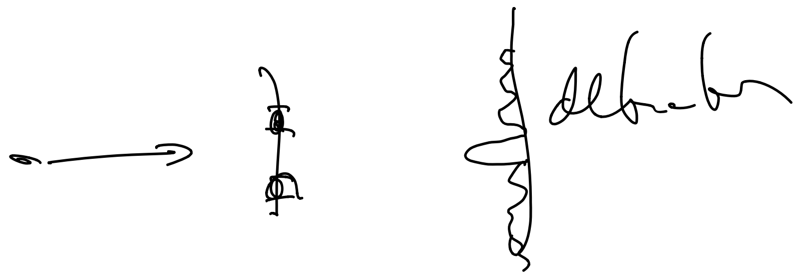


F → "Action"

space of states



evolve / time.



$$3\frac{1}{2}x^2+2x\Big]_2^3 = \frac{3}{2}3^2+2(3)-\frac{3}{2}2^2-2\cdot 2$$

$$= \frac{27}{2}+6-6-4$$

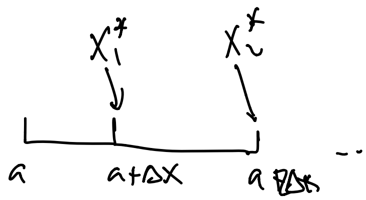
$$\frac{27}{2}-\frac{8}{2}=\frac{19}{2}$$

Use Riemann Sums and limits to find the value of the definite integral:

$$\int_2^3 (3x+2)\ dx$$

$a=2$ $\quad b=3$ $\quad n = \#$ rectangles

$$\Delta x = \frac{b-a}{n} = \frac{1}{n}$$



$$x_i^* = a + i\cdot\Delta x$$

$$= 2 + i/n$$

$$\text{ht}: f(x_i) = 3(2+i/n)+2$$

$$= 6 + 3i/n + 2$$

$$= 8 + 3i/n$$

Area: $\Delta x\, f(x_1^*) + \Delta x\, f(x_2^*) + \ldots$

$$= \sum_{i=1}^{n} \Delta x\, f(x_i^*)$$

$$= \sum_{i=1}^{n} \frac{1}{n}\left(8+3i/n\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n} 8+3i/n = \frac{1}{n}\sum_{i=1}^{n} 8 + \frac{1}{n}\sum \frac{3i}{n}$$

$$= \frac{8n}{n} + \frac{1}{n}\,\frac{3}{n}\underbrace{\sum_{i=1}^{n} i}_{\frac{n(n+1)}{2}}$$

$$= 8 + \frac{3}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= 8 + \frac{3}{2}\,\frac{(n+1)}{n}$$

$$= 8 + \frac{3}{2}\left(1 + \frac{1}{n}\right)$$

$$n \to \infty \implies 8 + \frac{3}{2}(1)$$

$$= \frac{19}{2}$$