MATH 2250 PRACTICE SHEET FOR FINAL EXAM

1. Use the definition of the derivative to find the derivative of the function
$$f(x) = x^2 + \frac{1}{x}$$

2. Find an equation for the tangent line to the graph of the function
$$f(x) = 3x + \ln x$$
at $x = 1$.
   Use this information to approximate $f(1.2)$.

3. Find the derivative of the function
$$f(x) = \frac{xe^x - 1}{\ln x}$$

deriv.

4. Solve for $\frac{dx}{dt}$ given the equation
$$\ln(x + y) = e^x - t$$

$\frac{d}{dt}$ both sides

$$\left(\frac{1}{x+y}\right)\left(\frac{dx}{dt}+\frac{dy}{dt}\right) = e^x\frac{dx}{dt} - 1$$

5. Compute the following limit
$$\lim_{x\to 0} \frac{\sin x^2}{\cos x - 1}$$

$$\left(\frac{1}{x+y}\right)\frac{dx}{dt} + \left(\frac{1}{x+y}\right)\frac{dy}{dt} = e^x\frac{dx}{dt} - 1$$

lim

6. Compute the following limit
$$\lim_{x\to 3} \frac{e^x - e^3}{x - 3}$$

7. Compute the following limit
$$\lim_{x\to 3} \frac{e^x - e^3}{x}$$

8. Find the absolute minimum and maximum values of the function $f(x) = x + \ln x$ on the interval $[1, e]$.

opt

9. Does the following function have an absolute maximum or absolute minimum value on the interval $[\frac{1}{2}, \infty)$?

$$f(x) = x - 7\ln x$$

FTC

10. Consider the function $f(x) = \dfrac{3}{1+x^3}$, and suppose that $F(x)$ is an antiderivative for $f(x)$ with $F(0) = 0$.

Explain why $F(x) = \displaystyle\int_0^x \frac{3}{1+t^3}dt$

RR

11. Two people start walking from the same point, person A walking due north and person B walking due east. After some time, if person A is 40 feet from the starting point and walking at 3 feet per second, and if person B is 30 feet from the starting point and walking at 5 feet per second, how fast is the distance between the two people changing?

12. Compute

$$\int e^x \cos e^x dx$$

13. Compute

$$\int (\sin x)^7 (\cos x) dx$$

S

14. Compute

$$\int_0^1 x\sqrt{1-x^2}\ dx$$

15. Compute

$$\int_0^1 \sqrt{1-x^2}\ dx$$

hint: this is a trick question

16. Compute

$$\int \tan x\ dx$$

(you shouldn't need to memorize this formula — use u-substitution!)

opt

17. Find two number a and b such that $3a + 4b = 9$ and such that ab is as large as possible.

18. Find all critical values of the following functions $x, x^{-1}, x^2, x^3, x^{2/3}, x^{-2/3}, x + \ln x$.
Which of these critical values represent local minimums and which represent local maximums?

$-x^{-2} = -\frac{1}{x^2}$

Σ

19. Use Riemann Sums with 3 rectangles and using left endpoints to approximate the value of the integral:

$$\int_0^1 \frac{1}{1+x^3}\, dx$$

20. Use Riemann Sums and limits to find the value of the definite integral:

$$\int_2^3 (3x+2)\ dx$$

opt

21. A company would like to design a box (bottom, top and four sides), with square base with a volume of exactly 1000 cubic centimeters. How tall should the box be made so that it uses the least amount of material (surface area)?

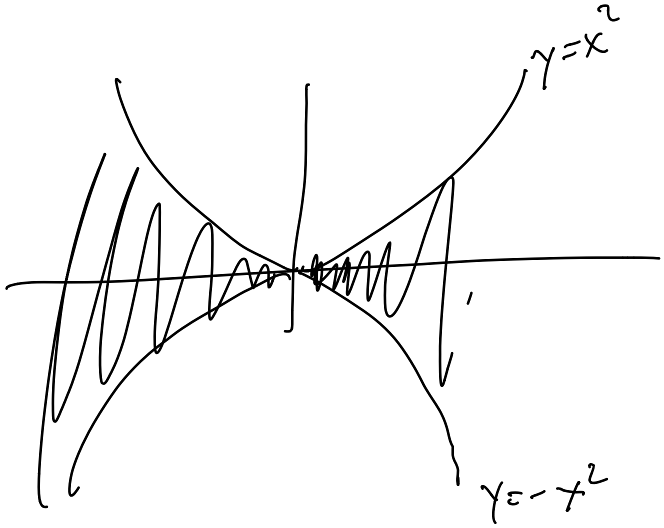
graph

22. Suppose that $f(x)$ is defined on $[-3, 3]$ which satisfies the following properties:
- $f(x)$ is increasing on the interval $[-3, 0]$,
- $f(x)$ is decreasing on $[0, 3]$,
- $f(x)$ is concave down on $[-3, 1]$, and
- $f(x)$ is concave up on $[1, 3]$.

Use this information to sketch the graph of $f(x)$.

23. Sketch a graph of a function which is increasing everywhere, concave down for $x < 0$ and concave up for $x > 0$.

$$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x} + \ln|x|\right)$$



$$\lim_{x \to 0} -x^2 = 0$$

$$= \lim_{x \to 0} x^2$$

$$-1 \leq \sin\left(\frac{1}{x} + \ln|x|\right) \leq 1$$

$$-x^2 \leq x^2 \sin(\quad) \leq x^2$$

$$\left(\frac{1}{x+y}\right)\left(\frac{dx}{dt}+\frac{dy}{dt}\right) = e^x\frac{dx}{dt} - 1$$

$$\left(\frac{1}{x+y}\right)\frac{dx}{dt} + \left(\frac{1}{x+y}\right)\frac{dy}{dt} = e^x\frac{dx}{dt} - 1$$

$$\left(\frac{1}{x+y}\right)\frac{dx}{dt} - e^x\left(\frac{dx}{dt}\right) = -1 - \left(\frac{1}{x+y}\right)\frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{-1 - \left(\frac{1}{x+y}\right)\frac{dy}{dt}}{\left(\frac{1}{x+y}\right) - e^x}$$

$$3\tfrac{1}{2}x^2 + 2x \Big]_2^3 = (3/2)3^2 + 2(3) - \left(3/2(2^2) + 2(2)\right)$$

$$= \frac{27}{2} + 6 - 6 - 4$$

$$\int_2^3 (3x+2)\ dx$$

$a=2$ $b=3$ n rectangles $\Delta x = \frac{3-2}{n} = \frac{1}{n}$

$x_1^* = a + 1\cdot\Delta x$ $x_2^* = a + 2\Delta x \ldots$

$x_i^* = a + i\Delta x = 2 + i\cdot\frac{1}{n}$

ht: $f(x_i^*) = 3(x_i^*) + 2 = 3\left(2 + i\cdot\frac{1}{n}\right) + 2$

Area: $\frac{1}{n}\left(3\left(2+1\cdot\frac{1}{n}\right)+2\right)+\frac{1}{n}\left(3\left(2+2\cdot\frac{1}{n}\right)+2\right)$

$+ \cdots$

$$= \sum_{i=1}^{n} \frac{1}{n}\left(3\left(2+i\frac{1}{n}\right)+2\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(6+\frac{3i}{n}+2\right) = \frac{1}{n}\sum_{i=1}^{n} 8+\frac{3i}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n} 8 + \frac{1}{n}\sum_{i=1}^{n}\frac{3i}{n}$$

$$= \frac{1}{n}8n + \frac{1}{n}\frac{3}{n}\underbrace{\sum_{i=1}^{n} i}_{\frac{n(n+1)}{2}}$$

$$= 8+\frac{3}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= 8+\frac{3n(n+1)}{2n^2} = 8+\frac{3n^2+3n}{2n^2}$$

$$= 8+\frac{3n^2}{2n^2}+\frac{3n}{2n^2}$$

$$= 8+\frac{3}{2}+\frac{3}{2}\frac{1}{n}$$

$$\lim_{n\to\infty} 8+\frac{3}{2}+\cancel{\frac{3}{2}\frac{1}{n}} = 8+\frac{3}{2} = 9.5$$

$$f(x) = \frac{\tan x}{x} \qquad g(x) = \frac{x^2}{xe^x - x}$$

$$h(x) = \frac{2x+1}{x^2+x} \qquad k(x) = \frac{\sin x}{x^2}$$