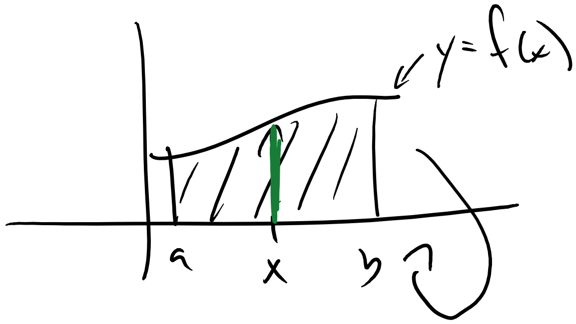
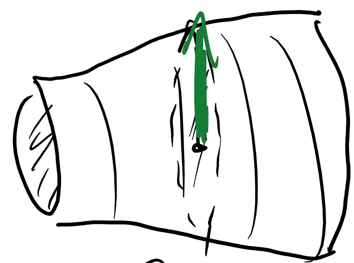
Lecture 5: solids of rotation

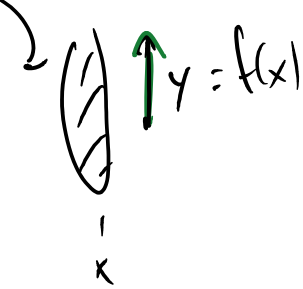
Wednesday, August 27, 2014 12:26 PM

Volumes of Revolution about x-axis





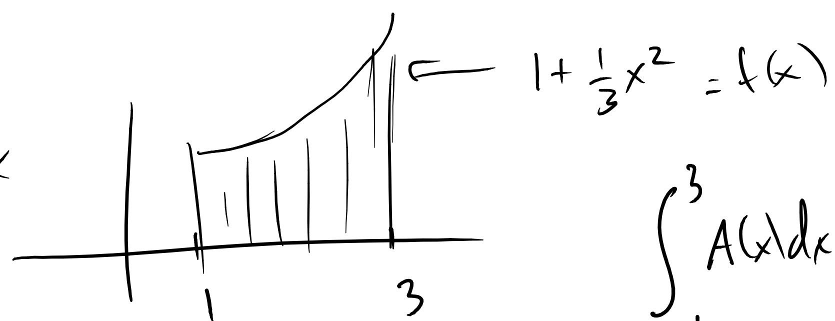
Volume



$$\int_a^b A(x)\,dx = \int_a^b \pi r^2\,dx \qquad r = y$$

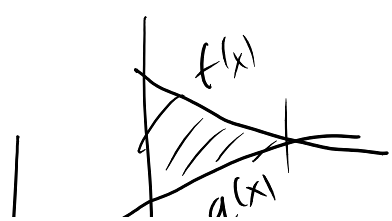
$$= \int_a^b \pi y^2\,dx = \int_a^b \pi f(x)^2\,dx$$

ex

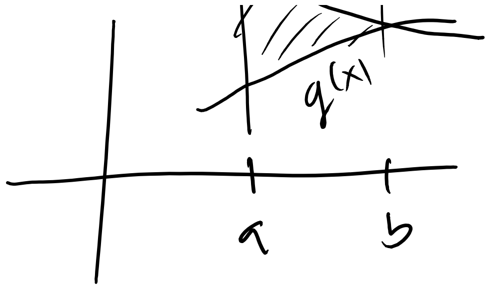


$$\int_1^3 A(x)\,dx = \int_1^3 \pi f(x)^2\,dx$$

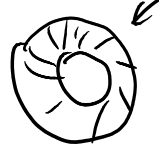
$$= \int_1^3 \pi \left(1 + \tfrac{1}{3}x^2\right)^2 dx$$



$$\int_a^b A(x)\,dx = \int_a^b \left(\pi f(x)^2 - \pi g(x)^2\right) dx$$



$$\int_a^b A(x)\,dx = \int_a^b \left(\pi f(x)^2 - \pi g(x)^2\right) dx$$



$A(x) = \pi f(x)^2 - \pi g(x)^2$

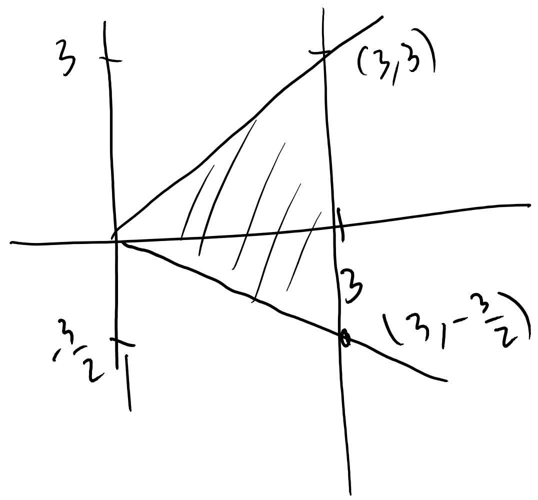
Practice 1. Find the volume of the solid generated by revolving the region described below about the y axis:

Region: enclosed by the lines $y = x$, $y = -\frac{x}{2}$, $x = 3$

Practice 2: Same, about x-axis

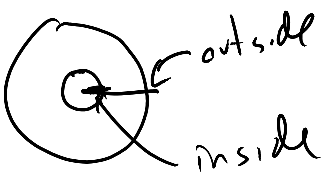
Region enclosed by $x = \sqrt{y}$, $x = -y$, $y = 2$

Prob 1



$$\int_{y=-\frac{3}{2}}^{y=3} A(y)\,dy$$

$$A(y) = \pi\,(\text{outside radius})^2 - \pi\,(\text{inside radius})^2$$



3!

changes!

top: $y = x$

bottom: $y = -\frac{1}{2}x$

$x = -2y$

$$\int_{y=-\frac{3}{2}}^{y=3} A(y)\,dy = \int_{y=-\frac{3}{2}}^{y=3} \pi 3^2 - \pi(\text{inside})^2\,dy$$

$$= \int_{y=-\frac{3}{2}}^{y=3} 9\pi\,dy - \pi \int_{y=-\frac{3}{2}}^{y=3} (\text{inside rad})^2\,dy$$

$$\left(\quad \right)^3 \quad -\pi ? = ??$$

$$= \left(9\pi y \right)\Big|_{y=-\frac{3}{2}}^{3} - \pi ? = ??$$

$$\int_{y=-\frac{3}{2}}^{y=3} (\text{inside})^2 \, dy = \int_{-\frac{3}{2}}^{0} (-2y)^2 \, dy + \int_{0}^{3} y^2 \, dy = ?$$

$$y = -\frac{1}{2}x$$