Personal Perspectives on the Mathematical Endeavor

Understanding, language, and agency in mathematical life

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CREDITS

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Mathematics has never been theorem production for me. The goal has always been understanding: understanding the world, understanding how people think about it, and finding new ways of seeing that make the mind itself feel different. The sign of success is not only that a problem has been solved. It is that something once difficult even to think about has become an ordinary part of the intellectual landscape.

AI does not change that aim. It changes the reach. Discovery and verification have acquired extraordinary new tools, and the mathematical world has become larger, faster, and more accessible. This is exciting because there is more available to understand.

The feeling of a brain changing

Understanding is often described as possession. A person understands a definition, knows a proof, has mastered a theory. The language suggests that knowledge has been acquired and placed somewhere safe, perhaps on a shelf between spectral sequences and the ability to assemble furniture without reading the instructions.

The more interesting experience is transformation.

A new subject first appears as resistant terrain. Its basic objects require effort to hold in mind. Every sentence has to be translated. A definition arrives, wanders off, and must be retrieved before the theorem containing it has reached the end of the page. Examples help, but initially they may feel like isolated acts of diplomacy with a country whose geography remains unknown.

Then the terrain changes. The definitions begin to support one another. Examples stop being anecdotes and become landmarks. A mechanism appears in several disguises. Connections that once required a deliberate chain of thought become available together. What

was difficult to think about is not necessarily easy, but it is inhabitable.

This is a peculiar kind of progress because the external mathematics may not have moved at all. The theorem is the same theorem. The definitions have not become shorter out of sympathy. What changed was the thinker.

The change can be felt in the questions that now occur without prompting, the structures that can be held together, and the judgments that become possible before a formal argument has begun. A field of intuition has expanded. The brain has received an update without providing release notes.

This is one reason learning and research cannot be separated cleanly. Research is not only the production of objects that did not previously exist. It is also the reorganization of thought around objects that were already there. Sometimes the decisive event is a new theorem. Sometimes it is an example that makes an old theorem visible for the first time. Sometimes it is a definition that causes three pages of difficulty to vanish and leaves everyone mildly offended that the definition had not arrived sooner.

The desired result is a larger intellectual landscape: more places from which to see, more paths between them, and more freedom to move without rebuilding the road at every step.

Proof is part of the landscape

Proof matters. It supplies public implication. It makes a claim answerable to something beyond confidence, status, or the persuasive use of a blackboard. A proof can be checked, criticized, repaired, transported, and eventually forgotten in enough detail that it must be checked again.

But proof is only one part of the landscape.

Questions, examples, language, interpretation, and connections do different work. A proof can show that a conclusion follows without showing why the statement was worth making. It can certify a formal proposition without settling whether that proposition faithfully expresses the human question. It can be correct and still fail to reorganize anyone's understanding.

The Hodge and Tate conjectures make the distinction vivid. It would matter enormously to know whether they are true. Their eventual proofs or counterexamples would be major mathematical events. Yet the ability to formulate such questions already represents a profound achievement. The conjectures require a conceptual world in which apparently different kinds of structure can be brought into relation and the missing piece can be seen clearly enough to be named.

A question does not float into mathematics as a grammatical sentence looking for an answer. It emerges from a way of seeing.

This matters in an age of inexpensive answers. An answer to a question nobody knows how to ask is not automatically part of human understanding. It may be correct. It may later provoke the invention of a question. But until there is a framework in which the answer can be interpreted, criticized, and used, its human meaning remains largely latent.

The theorem is important. So is the achievement of living in a world where the theorem can be wanted.

The cubic equation, twice encountered

Cardano's work on the cubic and quartic equations is astonishing partly because of the conceptual resources with which it was done. The solutions were developed in a world still strongly organized by Euclidean geometry, without negative numbers functioning as ordinary first-class objects in the modern algebraic sense.

The formulas were not waiting in contemporary notation. Cardano had to think his way through a landscape in which the distinctions between possible and impossible quantities, geometric magnitudes, and algebraic operations were arranged differently. The achievement belongs to that setting. Translating it backward into modern symbols can make the result look inevitable, which is a traditional way for later notation to accept credit for earlier courage.

Cardano did not then create the modern conceptual framework in which the work is now taught. The work was recast over time. Algebraic language changed. Negative and complex numbers acquired different status. Symbolic notation made operations available for sustained manipulation. The same result entered a different intellectual landscape.

The later framework did more than shorten exposition. It changed the experience of the mathematics. Relationships became easier to hold together. Operations that once required elaborate verbal management became visible on the page. Questions that would have been awkward to formulate became natural extensions of the language.

The result persisted. The experience of the result changed.

That distinction is easy to miss because modern formulations tend to present themselves as the mathematics finally removing its historical costume. They are better understood as powerful creations: ways of organizing thought that make some structures immediate, some questions natural, and some older difficulties nearly invisible.

The disappearance of a difficulty is not evidence that the difficulty was foolish. It may be evidence that a great deal of conceptual work has become infrastructure.

The pull of parallax

Mathematical objects do not appear from only one direction. A structure may be encountered through physics, geometry, algebra, arithmetic, computation, or a carefully chosen example that refuses to identify its department. Each perspective makes different features visible. Each has its own primitive objects, preferred mechanisms, and sense of what requires explanation.

There is a strong temptation to ask which perspective is the real one. This usually produces a lively discussion and no reduction in the number of perspectives.

The more productive attraction is parallax. In ordinary vision, the shift between viewpoints is a cue to depth. In mathematics, the noncoincidence of perspectives can play a similar role. Geometry may expose shape and locality. Algebra may expose operations and universal structure. Arithmetic may expose rigidity and discrete variation. Physics may expose mechanism, symmetry, and the unreasonable productivity of asking what the universe appears to be computing.

Somehow, at the bottom, these may be approaches to the same object. There is no need to settle in advance whether the underlying unity is ontological, structural, analogical, or a temporary convenience that survives because it keeps producing the right questions.

Reconciliation does not require flattening the views into one official language. A perspective can be understood without being dissolved. The goal is to inhabit several viewpoints, understand what each makes

natural, and feel their relationships as part of one intellectual landscape.

Perspectives belong to people too. One mathematician enters through an example. Another sees a universal property before breakfast and assumes the example will introduce itself later. A third recognizes a physical mechanism. A fourth hears an arithmetic analogy that nobody else hears until it becomes the organizing principle of the subject.

Understanding how different people understand the same object is not a detour from understanding the object. Their routes reveal different affordances and obstructions. Making those viewpoints personal enlarges the available mathematics without requiring any one of them to become sovereign.

No formulation is inevitable

There may be a deep structure of mathematics that is discovered. This phrase is allowed to remain mysterious, and we are not attempting to disambiguate or resolve it. Mathematics has spent centuries debating what its objects are and has shown no sign of allowing a short essay to settle the matter before lunch.

Whatever may be discovered, the languages of mathematics are created.

Notation, definitions, diagrams, metaphors, examples, terminology, and larger conceptual frameworks arise historically. They reflect purposes, available problems, inherited habits, and choices that could have gone differently. No formulation arrives with a certificate stating that it was the only language the underlying structure would permit.

This does not make mathematical language arbitrary. The mathematics pushes back. A notation can obscure essential dependence. A definition can group the wrong examples. A metaphor can be productive for a decade and then begin quietly generating false expectations. Created languages are constrained by what they are being used to understand, but constraint is not inevitability.

Language mediates mathematical experience dramatically. It affects what appears primitive or derived, simple or difficult, visible or hidden, connected or isolated. It influences which thoughts can be sustained long enough to become questions. A good language does not merely provide concise answers. It enlarges the space of askable things.

This is why exposition can change mathematics. An attempt to explain a theory may reveal that its basic definitions are fighting the governing intuition. A change of notation can turn a recurring proof into a single mechanism. A diagram can expose a symmetry that prose had been requiring everyone to reconstruct privately and at considerable personal expense.

Mathematical language is cognitive technology. It does not simply report what has been thought. It participates in making thought possible.

Conceptual co-creation

Created language does not imply clean authorship. The history of mathematics is not a sequence of isolated minds depositing complete conceptual systems into the literature. It is conversations, schools, borrowed metaphors, competing conventions, correspondence, teaching, errors, repairs, reinterpretations, and examples that acquired more authority than the people who introduced them expected.

Authorship has always been messy because intellectual development has always been collaborative.

The appropriate idea is conceptual co-creation. A framework develops through the interaction of people, traditions, artifacts, problems, and time. A definition may be named after one person and owe its usable form to twenty. A theory may be transformed by the person who proves its theorem, the person who finds its decisive counterexample, and the person who teaches it with the right picture.

AI can be included in this collaboration without apology or theatrical reluctance. It can generate examples, detect patterns, translate among formulations, search mathematical memory, test conjectures, propose language, formalize implications, and expose connections that no active participant was holding in view. These are not negligible contributions merely because their source is unfamiliar.

Nor does welcoming AI require declaring every generated connection profound. Human collaborators have already demonstrated that collaboration can produce irrelevance at scale. The usual standards remain: what does the contribution clarify, what supports it, what changes because of it, and can another participant inspect the route?

Conceptual co-creation shifts the emphasis away from exclusive ownership. The important question is not who can claim sole authorship of a framework. Such a person is usually imaginary. The important question is whether the participants have meaningful agency within its development.

Agency means active participation

Agency is sometimes reduced to keeping a human in the loop. Loops vary in quality. A conveyor belt is a loop if the factory is arranged

correctly.

Meaningful agency requires more than final approval. It includes choosing what deserves attention, shaping the question, evaluating whether an answer matters, redirecting the inquiry, modifying the tools, refusing their implicit goals, and allowing the encounter to change the purposes with which it began. The human participant is not merely an error detector attached near the end of the process.

Every technology is designed, explicitly or otherwise. Its defaults, interfaces, available actions, measures of success, and assumptions about the user have been chosen by someone: the user, another mathematician, an engineer at OpenAI, an institution, or an inherited convention whose authors have become difficult to subpoena. There is no neutral absence of design. There are only designs whose purposes and consequences are more or less visible.

This creates a fundamental distinction between active co-creation and commodity status. An active participant can steer, evaluate, modify, refuse, and be changed. A commodity supplies attention, behavior, work, or intellectual life to a system whose purposes are determined elsewhere.

The distinction is not human versus AI. It is participation versus being processed.

Nor is it a distinction between building every tool personally and using tools made by others. Mathematical life has always depended on inherited infrastructure. Agency can be collaborative. It can use systems built by thousands of people and still retain the ability to notice what those systems are optimizing, decide whether that optimization serves the inquiry, and reshape the relationship when it does not.

This is why human benefit cannot be specified entirely outside human

participation. Benefit is not a label attached by a sufficiently confident designer. People have to be present in the activity strongly enough to judge what it is doing to their questions, capacities, communities, and curiosity.

Agency is not an obstacle added to an otherwise efficient system. It is part of what the system is for.

The unchanged enterprise

AI supplies superpowered tools for discovery and verification. It enlarges the number of examples that can be explored, the arguments that can be tested, the literatures that can be compared, and the formulations that can be tried before one begins to settle. This makes the mathematical world richer and more accessible. It also makes that world capable of producing plausible material considerably faster than anyone can absorb it, which is abundance performing its customary trick of arriving with a new scarcity attached.

The scarce resource becomes integration. Which result changes a picture? Which connection deserves to reorganize a course? Which new formulation makes an old obstacle disappear for the right reason? Which machine-generated idea belongs in a human intellectual landscape rather than in a warehouse of implications?

The answer is not to slow discovery until human habits catch up. Nor is it to assign people the permanent role of checking machine work. Machines can already verify many things more reliably than people, and the category will expand. The deeper task is to develop the intellectual and cognitive technology that lets enlarged discovery become enlarged understanding.

That means new concepts, better languages, multiple entrance points,

learnable organizations of knowledge, and forms of collaboration in which contributions can transform the participants rather than merely accumulate around them. It means treating a theorem as one event in the development of a conceptual framework, not as the entire reason the framework exists.

The future mathematical question is therefore not which old task remains exclusively human. Exclusivity is a poor foundation for curiosity. The better question is what can become visible when people, agents, languages, and artifacts participate in conceptual co-creation without surrendering agency.

What can become natural? Which perspectives can be brought into productive relation? What language is missing? What kind of collaborator becomes possible?

A theorem can settle an implication. A language can open new worlds. Understanding makes these new worlds inhabitable.