If $U \xrightarrow{f} X$ is an fpqc cover

get an equiv. of categories between $QCoh(X)$

and descent data for

$QCoh(U \to X)$

look at full faithfulness

$$\mathrm{Hom}_{\mathcal{O}_X}(\mathcal{F}, \mathcal{F}') \overset{?}{=} \mathrm{Hom}_{(U \to X)}(\mathcal{F}_{U \to X}, \mathcal{F}'_{U \to X})$$

$$\begin{array}{ccc} \mathcal{F}_U & \xrightarrow{\varphi_U} & \mathcal{F}'_U \\ \swarrow & & \downarrow \\ \mathcal{F}_{U \times U} & \longrightarrow & \mathcal{F}'_{U \times U} \end{array}$$

descent data:

$$(\mathcal{G}_U, \pi_1^* \mathcal{G}_U \xrightarrow{\varphi} \pi_2^* \mathcal{G}_U)$$

$$\mathcal{F} \rightsquigarrow (\mathcal{F}|_U, \underset{\underset{\mathcal{F}|_{U \times U}}{\parallel}}{\pi_1^* \mathcal{F}_U} \xrightarrow{id} \underset{\underset{\mathcal{F}|_{U \times U}}{\parallel}}{\pi_2^* \mathcal{F}_U})$$

$$\mathrm{Hom}_{\mathcal{O}_X}(\mathcal{F}, \mathcal{F}') \to \mathrm{Hom}_{\mathcal{O}_U}(\mathcal{F}|_U, \mathcal{F}'|_U) \rightrightarrows \mathrm{Hom}_{\mathcal{O}_{U \times U}}(\mathcal{F}|_{U \times U}, \mathcal{F}'|_{U \times U})$$

Affine translation: $N = \mathrm{Hom}_{\mathcal{O}_X}(\mathcal{F}, \mathcal{F}')$ $\quad X = \mathrm{Spec}\, R \quad U = \mathrm{Spec}\, S$

$$N \to N \otimes_R S \rightrightarrows N \otimes_R S \otimes_R S$$

beginning of Amitsur complex

$$0 \to N \to N \otimes_R S \to N \otimes S \otimes S \to N \otimes S \otimes S \otimes S \to \cdots$$

$\mathcal{N}$ q.coh. sheaf $\longleftrightarrow N$

(on site) $\Gamma(\mathcal{N}) \to \Gamma(\mathcal{N}|_{\mathcal{U}}) \to \Gamma(\mathcal{N}|_{\mathcal{U}\times\mathcal{U}})$

$U_i \qquad U_i \cap U_j$

$\mathcal{U} \longleftrightarrow U_i$

Amcomplex is also $\check{C}(\operatorname{Spec} S \to \operatorname{Spec} R, \mathcal{N})$

$(\Rightarrow H^n_{et}(\operatorname{Spec} R, \mathcal{N}) = 0$ q.coh

$$0 \to N \to N \otimes_R S \to N \otimes S \otimes S \to N \otimes S \otimes S \otimes S \to \cdots$$

$\otimes S$

$$0 \to N \otimes S \to N \otimes S \otimes S \to \cdots$$

$n \otimes s_1 s_2 \qquad n \otimes s_1 \otimes s_2$

Clearly choose a series of these maps giving a homotopy $\mathrm{id} \sim 0$

$\Rightarrow$ exactness

Comparison of Zariski & étale

$\exists$ cont. map of sites $X_{\acute{e}t} \xrightarrow{\pi} X_{Zar}$

Groth. tops $X_{Zar} \xrightarrow{\pi^{-1}} X_{\acute{e}t}$

Leray spectral sequence:

$$H^p(X_{Zar}, R^q\pi_* \mathcal{F}) \Rightarrow H^{p+q}(X_{\acute{e}t}, \mathcal{F})$$

$(\pi^{-1})^*$

<u>idea</u>: compute $\Gamma_{\acute{e}t}$, can make Zar covs, refine w/ étale of each Zar open

$R^q\pi_*\mathcal{F}$

$\|$

Sheafification of

$(U \to X) \rightsquigarrow H^q_{\acute{e}t}(U, \mathcal{F})$

$$(R^q\pi_*\mathcal{F})_x = R^q\pi_*\mathcal{F}_x = R^q\Gamma(\mathcal{F}_x)$$

Base change to $X = \operatorname{Spec} \mathcal{O}_{X,x}$

$H^q_{\acute{e}t}(\operatorname{Spec}\mathcal{O}_{X,x}, \mathcal{F}_x)$

If $\mathcal{F}$ q-coherent, and X is reasonable enough that $\check{H} = H_{\acute{e}t}$

then, for U affine $H^q_{\acute{e}t}(U,\mathcal{F}) = \check{H}^q_{\acute{e}t}(U,\mathcal{F})$

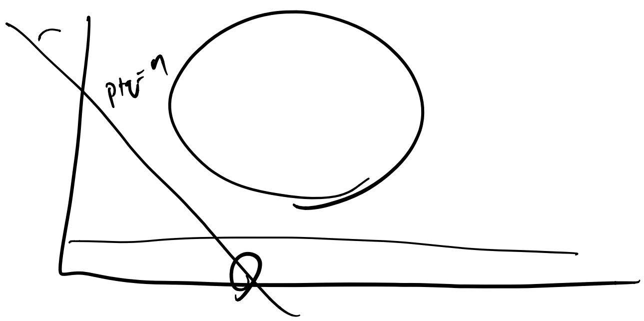
$= \lim \check{H}^q(V \to U, \mathcal{F})$

$= \lim H^q(\text{Amitsur complex})$

$= 0$ if $q > 0$

$$H^p(X_{Zar}, R^q\pi_*\mathcal{F}) = \begin{cases} 0 & \text{if } q > 0 \\ H^p(X_{Zar}, \overset{\mathcal{F}}{\overset{\shortparallel}{R^0\pi_*\mathcal{F}}}) & q = 0 \end{cases}$$

sheaf. for presheaf which
$U \to X \rightsquigarrow \mathcal{F}(U)$



<u>Thm</u> $H^n(X_{Zar}, \mathcal{F}) = H^n(X_{ét}, \mathcal{F})$

holds if X is <u>separated</u>. $\mathcal{F}$ q.coh.

we used (due to laziness)

Artin: $\check{H} = H_{ét}$ if X is q.c w/ every finite subset of pts in an affine.

Galois descent / cohomology

$\mathrm{Spec}\, E \longrightarrow \mathrm{Spec}\, F$

$$\begin{array}{c} E \\ |\, G \\ F \end{array}$$

Galois

$\mathrm{Spec}\, E \leftleftarrows \mathrm{Spec}\, E \otimes_F E$

$E \rightrightarrows E \otimes_F E$

$E = \dfrac{F[x]}{f(x)}$

roots are $\alpha_i \in E$

$E \rightrightarrows E \otimes_F E = \dfrac{E[x]}{f} \cong \prod \dfrac{E[x]}{(x-\alpha_i)} = \prod E$

$\alpha_1 \xrightarrow{1^{st}} \alpha_1 \otimes 1 \longmapsto (\alpha_1, \alpha_1, \ldots)$

$\xrightarrow{2^{nd}} 1 \otimes \alpha \longmapsto (\alpha_1, \alpha_2, \ldots)$

$(\alpha, \ldots \alpha)$

$(\sigma_1 \alpha, \sigma_2 \alpha, \ldots)$